On the planetary theory of sunspots

It has been proposed that sunspot activity is affected by positions of the planets, and calculations have been presented, which purport to show that planetary tides on the Sun vary in the same way as the sunspot variations. We believe that the apparent agreement of the sunspot cycle with planetary tidal effects is an artefact of the calculation.

The calculation in question was used to compute the absolute value of the difference in tidal potential between Earth–Venus conjunctions and oppositions at the sub-Jupiter solar point. The effect of Mercury, one of the strongest tide-raising planets, was ignored on the basis that its period is short compared with that of sunspot activity. The absolute value of the tide and the effect of partial line-ups of Venus (or the Earth) with Jupiter were not computed. Furthermore, it is not clear that the absolute difference between opposition and conjunction tidal potentials has any physical meaning.

Here we compute the full tidal problem for Mercury, Venus, Earth and Jupiter, the tide-raising planets, taking into account the complete orbital elements, including eccentricity, inclination and their variation with time.

At any given time, the tidal potential at a given point on the surface of the Sun is proportional to

\[ T = \sum \frac{m_i}{d_i^3} (\cos^2 z_i - \frac{1}{3}) \]

where \( m_i \) is the mass of the planet, \( d_i \) its distance from the centre of the Sun and \( z_i \) the angle from the planet to point \( M \) as seen from the centre of the Sun. We restrict ourselves to Mercury, Venus, Earth and Jupiter. Mars, Saturn and the other outer planets can easily be shown to have trivial contributions compared to the above planets.

In the plane of the ecliptic, the potential depends on the longitude \( \varphi \) through a first order polynomial of \( \sin 2\varphi \) and \( \cos 2\varphi \). Over a period of 10 d (our sampling interval) the Sun revolves about halfway around its axis and therefore any given point attached to its surface is subject to the whole variation of the tidal potential. So we characterised our problem by the maximum value (over \( \varphi \)) of this polynomial. We note that Mercury is the slowest planet around the Sun relative to a point on the Sun's surface and its contribution, contrary to previous arguments, should therefore not be neglected.

The positions of the planets were computed from the best available planetary elements for 65,536 epochs at intervals of 10 d, or roughly 1,800 yr, starting in the year 1800. A fast fourier transform (FFT) technique was then used to extract the power spectrum for comparison with the solar activity spectrum.

Figure 1 shows the tidal potential as a function of time for the years 1800–1808, both including Mercury (upper trace) and excluding it (lower trace). The high frequency effect is due to the eccentricity of Mercury's orbit and has a period of 0.24 yr.

Figure 2, an extension of the lower trace in Fig. 1, shows the tidal potential for the 25-yr period 1800–1825, excluding Mercury. The long period, beat-type phenomenon (\( \sim 11.9 \) yr) is due to the eccentricity of Jupiter's orbit. By contrast with the sunspot cycle, the tidal pattern repeats almost exactly every 11.9 yr and shows no evidence of a beat of \( \sim 100 \) yr; successive peaks in the tidal envelope are of almost exactly the same amplitude. Wood's samples (● in Fig. 2) have a spacing too large to provide a valid description of the tidal potential even excluding Mercury; such a sampling leads to aliasing of the lower frequencies. Figure 3 shows the power spectrum obtained from an FFT analysis of the whole 1,800-yr four-planet potential. The fundamental periods are those of the alignments of pairs of planets. Alignments of three or four planets come as beats of these primary values and consequently do not appear on the spectrum. The periods of Jupiter and Mercury and some of their harmonics show up because of the eccentricity of their orbits. The lower frequency part of the spectrum is very flat, Jupiter's being the longest period involved.

![Fig. 1 Tidal potential for the years 1800-08. a, Full four-planet tidal potential; b, tidal potential excluding Mercury. Both scales are identical.](image1.jpg)

![Fig. 2 Three-planet tidal potential for the years 1800-1825 (excluding Mercury). The marks (●) show the points used by Wood.](image2.jpg)
Fig. 3 Power spectrum of tidal potential. The horizontal axis is scaled as log$T$. The labels on the larger peaks identify the periods (yr) of alignments of planets (two-letter label) or the period of the planets themselves (one-letter label).

Fig. 4 Comparison of the lower frequency part of tidal spectrum (b) and sunspot spectrum (c; from ref. 5).

will be achieved in 1990. Even then, no special tidal effect occurs because alignment of the outer planets has no pronounced effects on the tides. Alignment of the tide-raising planets within 10 degrees is a common phenomenon, occurring approximately every 10.4 yr and is not associated with drastic tidal effects.

This research was supported by NASA.

Emile Okal
Don L. Anderson

Seismological Laboratory,
California Institute of Technology,
Pasadena, California 91125

Received October 31, 1974.
