III. MINERALS AND ROCKS

3.1 INTERPRETING SEISMIC VELOCITIES WITH DEPTH

The challenge before us is to interpret these curves in terms of the geology. What materials are these? To do this involves several different ideas:
1) Use the chemistry of the earth’s crust, which we can observe, to make useful extrapolations with depth.
2) Use the chemistry of the solar system (sun, planets, meteorites, etc...), to tell us about the material at depth in the earth.

There are two basic possibilities for structure in the deep earth (both will be discussed more later):
1) phase boundaries - material at depth is chemically the same as the material above it, but is transformed to denser phases (forms).
2) chemical boundaries - major compositional changes.

Each boundary - Moho, 410 km, 660 km, CMB, etc... may be either, and various lines of evidence need to be studied to determine their nature.

To preview our current, general view.
- The Moho is considered a chemical boundary so the mantle is somewhat chemically different from the crust.
- The 410 and 660 km discontinuities are thought to be primarily phase changes to denser forms.
- The core-mantle boundary (CMB) is almost certainly a chemical boundary from a “rock-like” mantle (we’ll define this later!) to an iron-nickel and a light element metal. The iron-nickel metal is much denser and allows the generation of a magnetic field.
- Almost nothing is known about the inner core. It’s also thought to be mostly iron-nickel, but is probably somewhat different from the outer core (its solid after all!)

Note: Our current, general view is subject to change without notice!
So, before starting on the chemistry, we need to say a bit about pressure.
3.2 PRESSURE AND DENSITY

Clearly, the question of the nature of a material is related to the pressure at some depth. The pressure at some depth is just the weight of the material above.

Units of pressure are bars = force/area.
1 bar = $10^6$ dynes/cm$^2$ = $10^5$ N/m$^2$ or Pascals (Pa)
(atmospheric pressure is 1 bar)

So near the surface $P = g \rho z$ where
- $\rho$ is density,
- $g$ is acceleration of gravity,
- and $z$ is depth

As an example, consider:
- $\rho = 3$ g/cm$^3$ (a good surface value)
- $g = 980$ cm/s$^2$
- $z = 3$ km = $3 \times 10^5$ cm

$P = (3 \times 10^5)(3)(980) = 8.8 \times 10^8$ dynes/cm$^2$ ~900 bars

A good rule of thumb is 3km ~ 1kbar

This is slightly trickier at depth since the acceleration of gravity changes with depth. However, it turns out (we won’t prove this) that only the material below a given depth matters

$$g(r) = \text{acceleration due to material below} = \frac{G \rho M}{r^2}$$

so we have

$$\frac{dP}{dr}(r) = -g(r)\rho(r)$$

Where the minus sign is because pressure decreases with radius (increases with depth)

For a homogeneous planet ($\rho$ constant, or an average value $\bar{\rho}$)

$$\frac{dP}{dr} = -\bar{\rho} \frac{G M(r)}{r^2} \left[ \frac{4}{3} \pi r^3 \right]$$

$$= -G\bar{\rho}^2 \frac{4}{3} \pi r$$
\[ P(r) = 2 \frac{\pi G}{3} \rho^2 (a^2 - r^2) \leftarrow \text{we choose a constant of integration make } P(a) = 0 \]

So for
\[
\begin{align*}
    a &= 6371 \text{ km, } \bar{\rho} = 5.5 \text{ g/cm}^3, \text{ at the center } (r=0) \\
    G &= 6.67 \times 10^{-8} \\
    P &= 1.7 \times 10^{12} \text{ dynes/cm}^2 = 1.7 \times 10^6 \text{ bars} = 1.7 \text{ megabar!!} \\
\end{align*}
\]

In reality, the core is much denser than the average and pressure at the earth’s center is closer to 3.5 Mbar.