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# EFFECT OF VARIABLE BATHYMETRY ON THE AMPLITUDE OF TELESEISMIC TSUNAMIS: A RAY-TRACING EXPERIMENT

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*Abstract.* We apply surface wave ray-tracing techniques to tsunami propagation in an ocean of variable depth. Because tsunami velocities are proportional to the square root of the depth of the water column, the juxtaposition of oceanic plateaux and basins generates strong lateral velocity gradients, and bathymetric features can act as lenses focusing or defocusing the tsunami waves. We present a reconstruction of the tsunami wave field from the great Chilean earthquake of 1960: an order-of-magnitude computation based on the concept of ray-density provides qualitative agreement with observations at several sites in the Pacific Ocean. We also demonstrate the possibility of using "reciprocal" maps to identify tsunamigenic zones that present particular danger for a given receiving shore.

## Introduction

The past several years have seen significant progress in resolving the gross structures of the Earth's lateral heterogeneity, in particular its effect on surface wave propagation. Ray-tracing experiments have clearly demonstrated that lateral velocity gradients can strongly focus and defocus Rayleigh waves, thereby forming caustics and gaps in the wave field, and affecting their amplitude as well as their phase [Lay and Kanamori, 1985; Schwartz and Lay, 1985; Tajima and Garmany, 1987]. At the low frequencies characteristic of mantle waves, lateral heterogeneity reaches $\pm 0.08$ km/s or $\pm 1.5\%$ of the mean phase and group velocities [e.g., Woodhouse and Dziewonski, 1984]. At higher frequencies, the effect of lateral heterogeneity on Rayleigh waves increases rapidly: in the Pacific Ocean and for waves of 20—40 s period, Yomogida and Aki [1987] inverted amplitude anomalies and found variations of typically $\pm 2\%$ in phase velocity. In the even more drastic case of the Arctic continental margin of North America, lateral heterogeneity reaches $\pm 5\%$ for 20-s Rayleigh waves, resulting in considerable amplitude anomalies across the U.S. for records of Novaya Zemlya explosions [Zeng et al., 1986].

The phase velocity of a tsunami propagating in an incompressible ocean is given in the long wavelength approximation by $C = \sqrt{gH}$ , where $g$ is the acceleration of gravity, and $H$ the depth of the water column. Since $H$ is known to vary substantially from young, broad, elevated spreading centers ($H \simeq 2.5$ km), to deep abyssal plains ($H \simeq 5.5$ km), the phase velocity is expected to similarly increase from $C \simeq 160$ m/s at the ridges to $C \simeq 230$ m/s in the oldest basins. This alone represents a $\pm 15\%$ deviation about the mean propagation velocity. Any island swell or oceanic plateau (e.g., Hawaii, Ontong-Java) will further decrease the local tsunami velocity. This order-of-magnitude computation leads us to expect focusing effects at least comparable in magnitude to those for 20-s Rayleigh waves. They could by themselves make the difference between a benign and a catastrophic tsunami.

This paper presents the result of a preliminary application of surface-wave ray-tracing techniques to the problem of tsunami propagation in the presence of lateral heterogeneity. Although variations in tsunami velocities have long been used for the accurate prediction of tsunami travel-times, and with the exception of some early work by Miyoshi [1955], little attention has been paid to its effect on amplitudes.



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## Method

### *Mapping Tsunami Velocities*

For this proof-of-concept experiment, we digitized the bathymetry of the Pacific Ocean Basin at 3° intervals from the Circum-Pacific map series [American Association of Petroleum Geologists, 1982], and converted the depths into velocity, using $C = \sqrt{gH}$ . In view of the long wavelengths ($\simeq 300$ km) characteristic of transpacific tsunamis, we then smoothed the velocity field with a 9-node rectangular operator that replaced the central value with the unweighted mean of the nine values. This minimized the effect of any exceedingly large or small values present. Finally, at such wavelengths, the phase and group velocities $C$ and $U$ are not significantly different, and we made their two fields formally identical.

The resulting field is illustrated in Figure 1, in which we have subtracted the mean velocity ($C_0 = 200$ m/s) from all values and contoured the velocity differential $C - C_0$ in units of 10 m/s. Note that despite the rather large grid interval and subsequent smoothing, the velocity field still reflects unambiguously the large scale bathymetry. Prominent on this figure are the shallow East Pacific Rise — Galápagos Rise — Chile Rise system, and to the West, the Tuamotu plateau. Similarly, the Hawaiian swell is recognized as a zone of locally deficient tsunami velocity. In the southwest Pacific, the complex regions of the Ontong-Java plateau, Fiji plateau, Lord Howe rise, Chatham rise, and Campbell plateau combine to form a zone of generally low velocities, and of strong velocity gradients. It is noteworthy that trenches, which are linear and narrow features, have little influence on $C$ since their effect is tapered by the filtering (or in real life by the long wavelength of the tsunami).

### *Ray-Tracing*

Julian [1970] first gave the solution of the eikonal equation in spherical coordinates, and Sobel and von Seggern [1978] applied it to the problem of ray-tracing on the surface of a laterally heterogeneous sphere in the case of a frequency independent phase velocity $C$. Although this formalism could be applied to tsunamis, we use the more complete solution of Jobert and Jobert

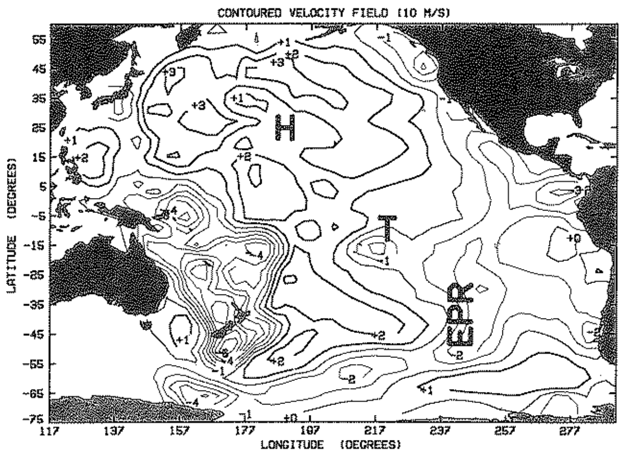


Fig. 1. Map of tsunami velocity field used in the present experiment. Contoured is the deviation of the velocity from a reference value of 200 m/s, in units of 10 m/s (e.g., +3 stands for 230 m/s; -2 for 180 m/s). Positive (fast) contours are bold; light ones negative (slow). Note low velocity regions at the East Pacific Rise (EPR), Tuamotu plateau (T), and (locally) Hawaiian chain (H).

[1983], who recast the equations in terms of group velocity $U$ to account for dispersion, and converted them into Cartesian coordinates via the Mercator transformation. It is straightforward to integrate the governing equations, in either formalism, with a fourth order Runge-Kutta scheme step-wise from the source outward to any distance. For this application, we terminate each ray as it reaches a shoreline.

## Results

We have reconstructed tsunami wave fields for several large historical earthquakes, and present in Figure 2 the resulting pattern in the geometry of the 1960 Chilean event, which offers some of the most striking effects.

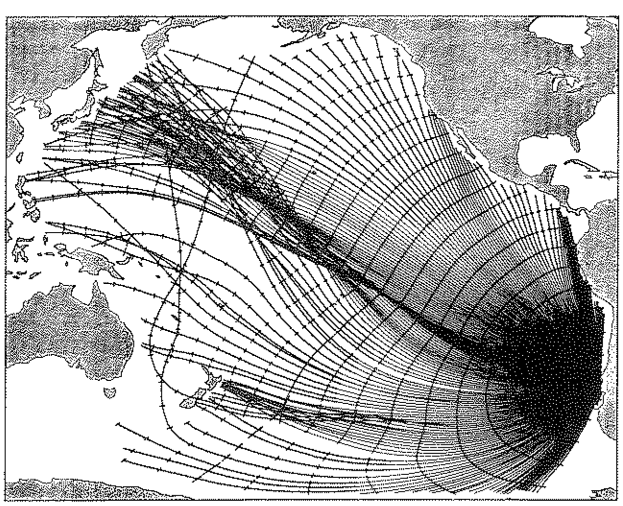

Fig. 2. Tsunami wave field resulting from ray-tracing for the 1960 Chilean earthquake. Rays are traced in 1° azimuth increments from the epicenter, and terminated upon reaching a continental shore. Tick marks show group times in increments of hours.

The most salient feature of this wave field is the region of strongly focused energy that extends northwest across the Pacific towards the Japanese and Kurile coasts. This represents the combined effect of the East Pacific Rise, basically acting as a converging lens, and of the Chile Rise and Tuamotu plateaux, which further act as waveguides to and from the lens. Smaller but significant caustics are also found along the coasts of Costa Rica and Nicaragua, at North Island, New Zealand, and weakly along California and the Baja peninsula.

On the other hand, large gaps exist in the ray field where substantial defocusing takes place. Prominent among these regions are the northeast rim of the Pacific basin, from British Columbia to the Aleutian island chain, and the entire westcentral Pacific, from the southern Mariana islands to the northern coast of New Zealand.

### *Preliminary Quantification*

In order to quantify satisfactorily the amplitude anomalies predicted by the ray-tracing, it will be necessary to compute synthetic marigrams in the presence of lateral heterogeneity, for example through a Gaussian beam technique [Červený et al., 1982; Yomogida and Aki, 1985]. In this preliminary study, we use the concept of ray density: following Lay and Kanamori [1985], we simply compute the linear density $D$ of rays per unit length of wave front in the vicinity of the receiver, and take the amplitude $a$ as proportional to $\sqrt{D}$ . This argument expresses conservation of energy within ray tubes, and correctly predicts $a \sim 1/\sqrt{\sin\Delta}$ in the case

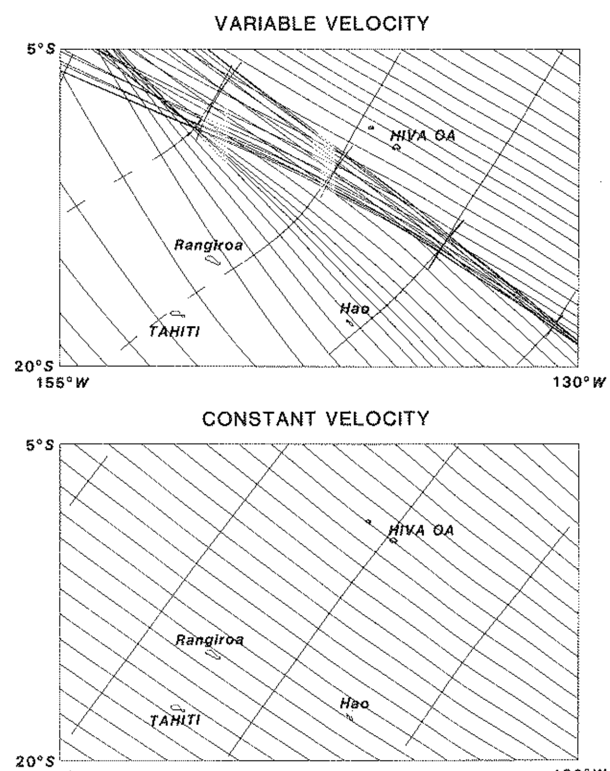


Fig. 3. *Top:* Close-up of Fig. 2 in the area of French Polynesia. Note defocusing at Tahiti, and closeness of Marquesas to caustic. *Bottom.:* Similar diagram for a homogeneous velocity field.

TABLE 1. Predicted amplitude anomalies due to focusing

| Receiving Shore† | $A$ |
|---|---|
| 1960 Chile | |
| Sendai, Japan | 1.1 |
| Hilo, Hawaii | 0.8 |
| Papeete, Tahiti | 0.8 |
| Hiva Oa, Marquesas | 1.8 |
| Reciprocal Diagram: Papeete | |
| Sendai | 0.4 |
| Kamchatka | 1.6 |
| Alaska | 0.6 |
| Reciprocal Diagram: Hilo | |
| Sendai | 2.3 |
| Kamchatka | 0.8 |
| Alaska | 0.9 |
| Macquarie Ridge | 2.6 |

† Epicentral area for reciprocal diagrams.

of a homogeneous velocity field. We obtain $D$ by simply counting the number of rays reaching the wavefront within one wavelength of the receiving shore. We then carry out the same investigation in the case of a homogeneous velocity field, and define the predicted amplitude anomaly as $A = [D_{actual} / D_{homog.}]^{1/2}$. Figure 3 illustrates the concept. We do not use the more sophisticated computation of the local ray intensity [Sobel and von Seggern, 1978], since it is not applicable in the vicinity of caustics, frequently present in our case.

In trying to compare the above results with actual report of amplitudes of the 1960 Chilean tsunami, one faces the classical problem of the separation of path and receiver effects. Specifically, the amplitude recorded at any given shore is controlled not only by the high-seas amplitude of the tsunami, but also by so-called "run-up" effects due to the interaction of the wave with continental shelves or island underwater structures, and to resonance of bays and harbors. Furthermore, directivity effects due to rupture propagation at the source create destructive interference in the direction along strike [Ben-Menahem and Rosenman, 1972]. In view of this difficulty, we restrict our computations to three locations: along the coast of Sendai in Japan, in the Hawaiian islands, and in French Polynesia. From the epicenter, these three areas are at an azimuth grossly orthogonal to the rupture, and thus directivity effects are expected to be negligible. Our results are summarized in Table 1.

*Japan.* Although Japan and the Kuriles lie at the extremity of the path of intense focusing by the East Pacific Rise and the Tuamotu plateau, they feature only an amplitude anomaly of 1.1 . This is because these shores are clearly beyond the caustics, and by the time the rays reach Japan, they have started to defocus again. Also, because of the extreme epicentral distance (about 155°), the effect of geometrical spreading is actually to increase amplitudes in the case of the homogeneous velocity field ; thus the origin of the catastrophic nature of the 1960 tsunami in Japan (200 people killed) is due primarily to its near-antipodal location, and not significantly to focusing.

*Polynesia.* Our result for Tahiti is $A = 0.8$ . The island lies in a region of defocusing to the South of the caustic. Despite the large amplitude of the 1960 tsunami (3 m at Papeete where run-up and resonance are negligible), it was deficient in relation to its huge seismic moment ($2\times10^{30}$ dyn-cm) by a factor of 2—3 when compared to smaller events such as the 1985 Mexican and 1986 Aleutian earthquakes [J. Talandier, pers. comm., 1986]. While the two numbers (observed and predicted) are not exactly equal, the sense of the anomaly is correctly predicted. In the Marquesas Islands, the effects of the tsunami were particularly devastating [Vitousek, 1963]. Because these are high islands not protected by coral reefs, run-up and resonance are expected to be significant. Nevertheless, our data show that they lie at a distance from the caustic comparable to the longest wavelengths for teleseismic tsunamis ($\simeq$ 450 km), and suggest a generally positive amplification factor at this site ($A = 1.8$).

*Hawaii.* Our calculations indicate defocusing ($A = 0.8$) at Hilo. Although the 1960 tsunami was catastrophic at Hilo [Eaton et al., 1961], the heights reached in the Hawaiian islands (at most a few m) are again not in relation to the earthquake's moment when compared to recent and smaller events (e.g., Aleutian, 1986).

*Seismic Reciprocity and Reciprocal Diagrams*

Figure 4 presents an interesting and potentially important application of these studies. It is the tsunami wave field generated by a hypothetical earthquake at Papeete, Tahiti. Using the concept of seismic reciprocity [Aki and Richards, 1980], we interpret the location of caustics as that of source regions for which focusing effects enhance tsunami amplitudes at Papeete. In this sense Figure 4 is a reciprocal map of tsunami enhancement due to lateral heterogeneity. In this particular case, the regions of particular "danger" are the Kamchatka peninsula, the Philippines, and certain sectors of the South American coast. On the other hand, Japan and the Kuriles, the Eastern Aleutians and Alaska are regions whose tsunamis are expected to be defocused. Additional epicenters in the Solomon—Vanuatu and

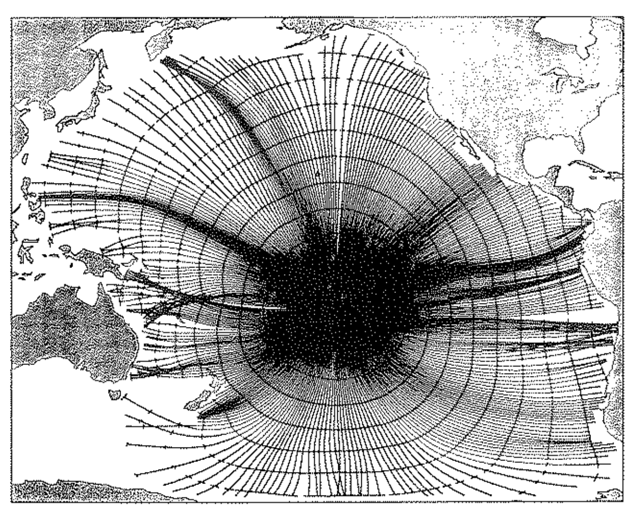

Fig. 4. Reciprocal diagram for Papeete, Tahiti. By virtue of the seismic reciprocity concept, this diagram (similar to Figure 2 for a hypothetical earthquake in Tahiti) predicts the amplitude at Papeete of tsunamis originating at a variety of epicenters.

Tasman Sea areas are on the dangerous list, but are not known to feature the gigantic events posing substantial tsunami risk. Table 1 shows that the effect of focusing varies by a factor of 4 between Japan and Kamchatka.

In the case of Hilo, our results suggest the opposite picture, i.e., amplification by a significant factor from the Sendai coast, and lower-than-normal amplitudes from Kamchatka. Finally, the case of Macquarie Ridge is of particular interest. For a homogeneous velocity model, this area is shadowed by New Zealand, both from Hawaii and Tahiti. For the laterally heterogeneous model, one predicts strong amplitudes at Hilo (our value in Table 1 was obtained by tracing the rays through New Zealand in the homogeneous model). While it is not a center of catastrophic seismicity, Ruff and Cazenave [1985] have shown that there exists the potential for significant subduction in the area.

## Conclusion

Our ray-tracing experiments clearly show the importance of focusing and defocusing by strong velocity gradients in controlling the amplitude of a teleseismic tsunami. While these results are fundamentally of a qualitative nature, gross order-of-magnitude computations based on the concept of ray density indicate that such effects can easily contribute a factor of 2 (either multiplicative or divisive) to the final tsunami amplitude. They clearly point our to the paths Sendai—Hawaii, Kamchatka—Tahiti and Central Chile—Marquesas as featuring focusing, and thus carrying enhanced tsunami hazard. It is clear that such results call for the systematic study of tsunami propagation in the Pacific (and possibly in other oceans as well) by means of a complete quantitative method, such as Yomogida and Aki's [1985] adaptation of the Gaussian beam technique [Červený et al., 1982]; this will be the subject of a future investigation.

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