Ultra-long period seismic moment of the great December 26, 2004 Sumatra earthquake and implications for the slip process

Seth Stein and Emile Okal
Department of Geological Sciences, Northwestern University, Evanston IL 60208

Abstract

Analysis of the longest period normal modes of the Earth, excited by the December 26, 2004 Sumatra earthquake yields a moment of 1.3×10^{30} dyn-cm, approximately three times larger than the 4×10^{29} dyn-cm measured from long-period surface waves. Hence the earthquake's ultra-long period magnitude, $M_w = 9.3$, is significantly larger than the previously reported $M_w = 9.0$, making the earthquake the second largest ever instrumentally reported. The higher magnitude presumably reflects slow slip not detected by the surface waves. Although the mode data do not constrain the location of the slow slip, a likely explanation is that it occurred over the entire 1200-km length of the rupture zone shown by aftershocks, of which only about the southern 1/3 to 2/3 appears to have slipped based on body-wave slip inversions. If so, then accumulated strain on the northern part of the rupture zone has also been released, leaving no immediate danger of a large tsunami being generated by slip on this part of the plate boundary.

These results come from analyzing the normal mode multiplets ${}_0S_2$, ${}_0S_3$ and ${}_0S_4$, with periods of about 3231, 2134, and 1546 s, respectively. The multiplets consist of 5, 7 and 9 singlets or peaks, respectively, which are split, *i.e.,* have distinct periods or eigenfrequencies, due to the rotation and ellipticity of the Earth. Great earthquakes like the Sumatra earthquake excite these long period multiplets sufficiently that they can be observed by Fourier analysis of long seismograms (Figure 1). Complementary data was obtained from the [unsplit] radial modes ${}_0S_0$ and ${}_1S_0$ with periods of 1227 and 613 s, respectively†.

The singlets can be described by their spectral amplitude, attenuation, and eigenfrequency. The amplitude of each singlet depends on the location of the earthquake and seismic station, earthquake depth and focal mechanism, and seismic moment [*Stein and Geller*, 1977]. The decay of energy with time, or equivalently the width of the spectral peak, depends on the mode's quality factor Q, a measure of attenuation. The singlet eigenfrequencies have been calculated by *Dahlen and Sailor* [1979].

Using the focal mechanism from the Harvard CMT project (strike 329°, dip 8°, slip 110°) and a focal depth of 15 km, we used two methods to estimate the seismic moment M_0 and Q for ${}_0S_2$, ${}_0S_3$ and ${}_0S_4$ at seven, five and four stations, respectively. One consisted of fitting the amplitude spectra in the frequency domain (Figure 1). A second

† The study of ${}_0S_0$ is preliminary since the proper analysis of its spectral line requires in principle a time series lasting at least 82 days (the product of its period by its quality factor).

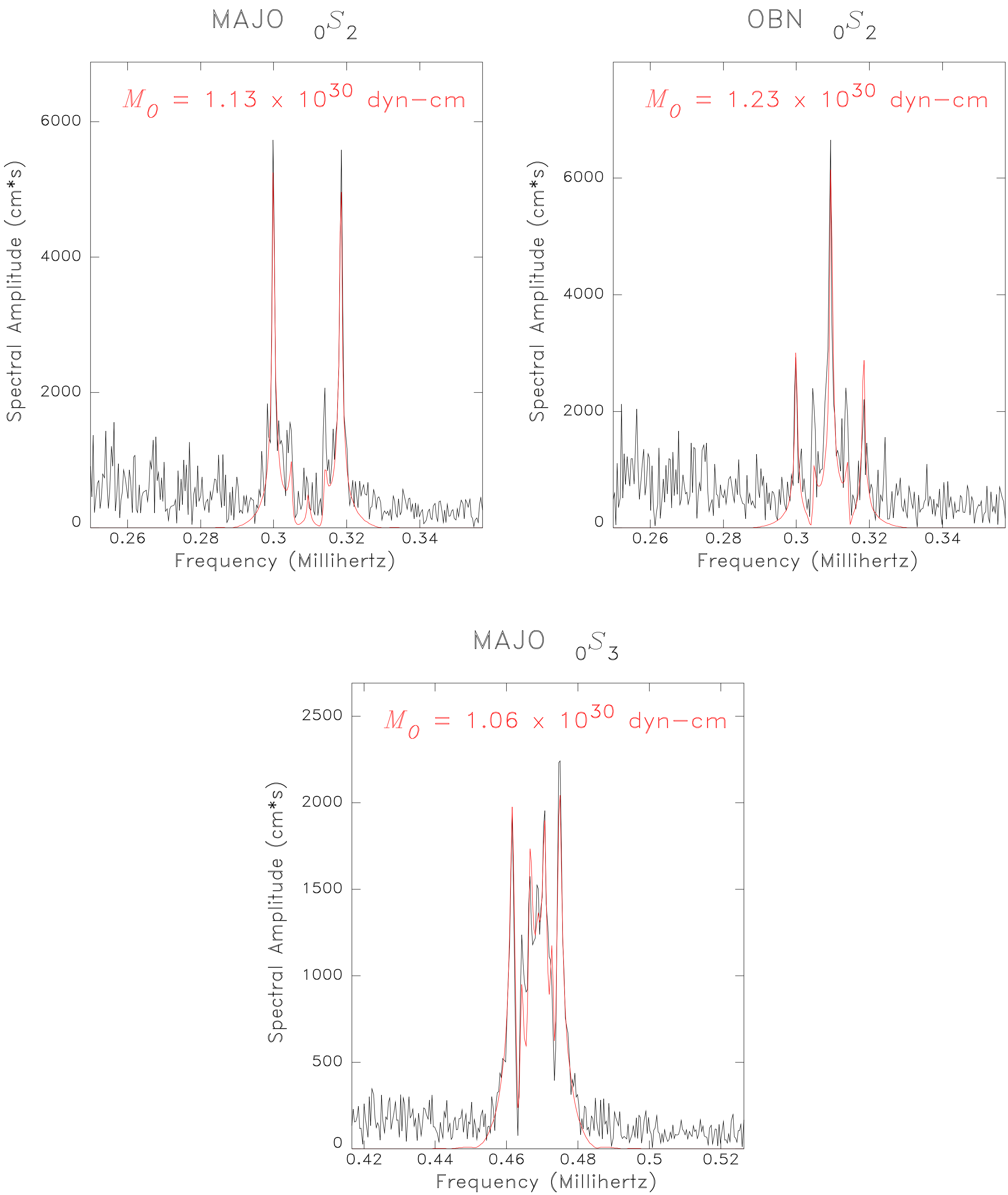


Figure 1.: Observed (black) and predicted (red) amplitude spectrum for the $_0S_2$ and $_0S_3$ multiplets, showing best-fitting seismic moment.

involved narrow-band filtering isolated singlets in the time domain, computing the envelopes of the decaying time series using the Hilbert transform, and fitting the logarithm of the envelope to estimate Q [*Geller and Stein,* 1979] and the moment from the extrapolated initial amplitude at the earthquake origin time (Figure 2).

These approaches yield consistent estimates of the moment and Q. For ${}_0S_2$, we find M_0 averaging 1.3×10^{30} dyn-cm and Q averaging 560; for ${}_0S_3$ we find M_0 averaging 9.5×10^{29} dyn-cm and Q averaging 445; for ${}_0S_4$ we find M_0 averaging 8.9×10^{29} dyn-cm and Q averaging 360. These Q estimates are consistent with previously reported values [*Stein and Nunn,* 1981; *Tanimoto,* 1990].

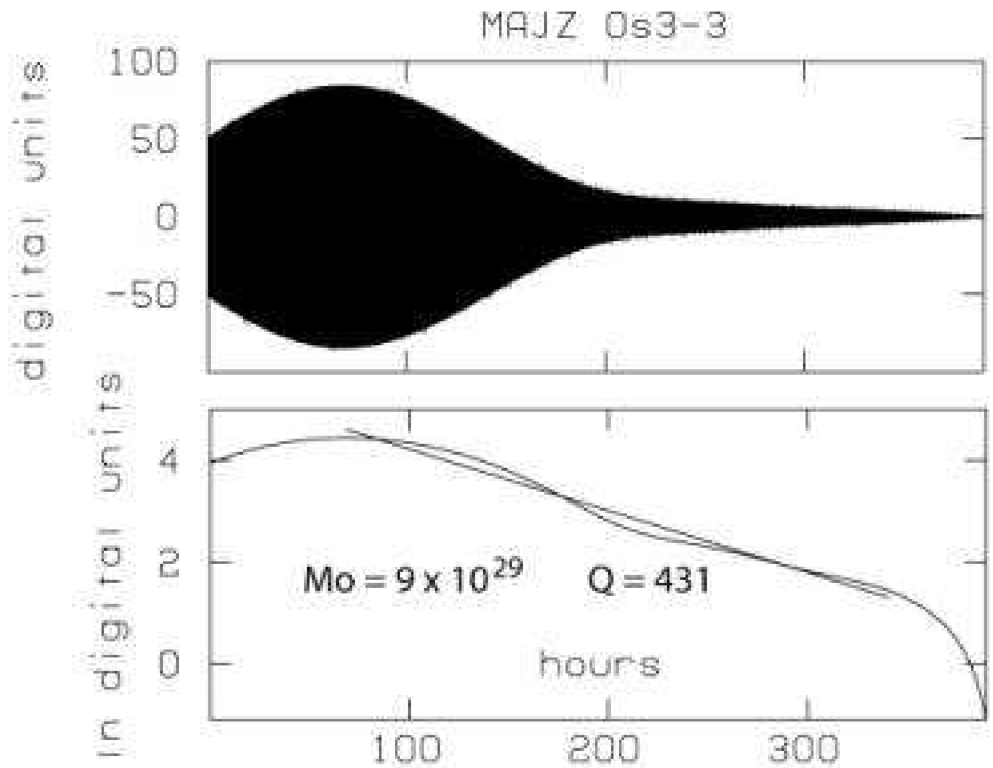
In the case of the radial modes ${}_0S_0$ and ${}_1S_0$, we constrained Q to its published values (5700 and 2000, respectively), and obtained moments of 8.4×10^{29} and 5.0×10^{29} dyn-cm, respectively.

These moment estimates for the gravest modes are 2 to 3 times larger than the 4×10^{29} dyn-cm measured from 300−s surface waves. Hence the earthquake's ultra-long period magnitude, $M_w = 9.3$, is significantly larger than the previously reported $M_w = 9.0$. This makes the earthquake the second largest ever instrumentally reported, larger than the 1964 Alaska earthquake ($M_0 = 8.2 \times 10^{29}$) but smaller than the 1960 Chile earthquake ($M_0 \geq 2 \times 10^{30}$ dyn-cm), assuming that these events' reported moments do not also underestimate their true size.

The moment estimates show a remarkable increase with increasing period (Figure 3), a property hitherto unreported for other events at such ultra-long periods. It raises the intriguing question of the period at which the source size actually ceases to grow. This source behavior presumably reflects slow slip not detected by the surface waves used in such algorithms as Harvard's CMT project.

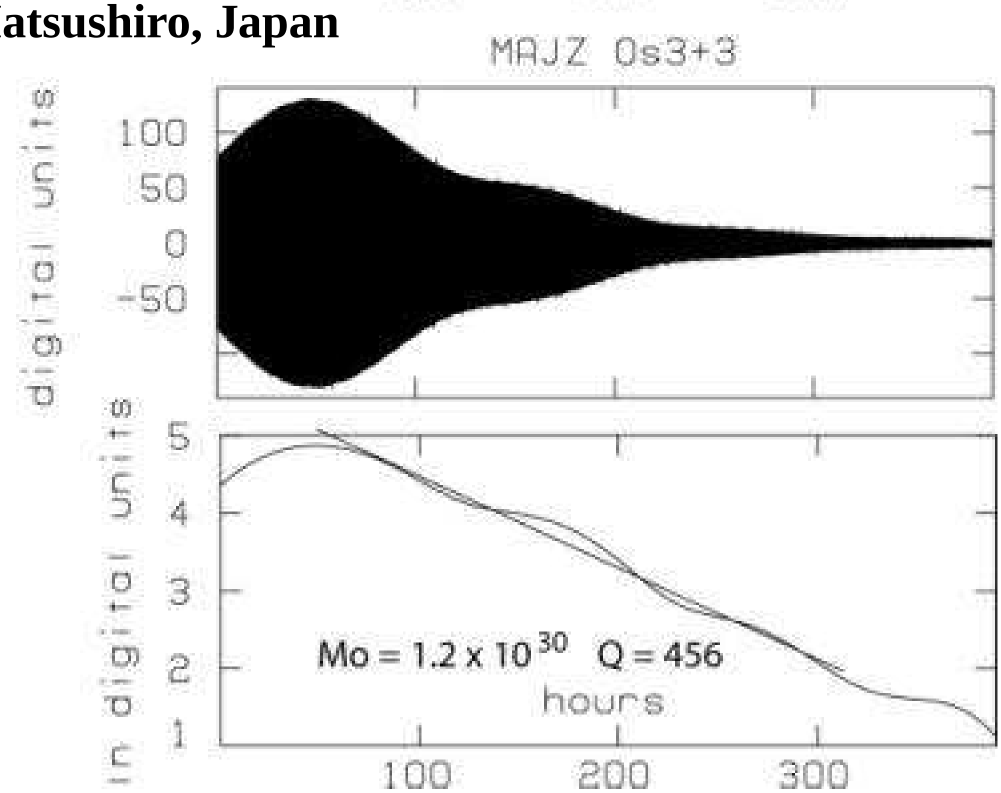
Although the mode data do not constrain the location of the slow slip, a likely explanation is that it occurred over the entire 1200-km length of the rupture zone shown by aftershocks (Figure 4). For example, assuming a rigidity of 4×10^{11} dyn/cm^2, 13 m of slip on a fault 1200 km long and 200 km wide (down dip dimension) implies a moment of about 1.3×10^{30} dyn-cm. This would be a larger fault area than implied by body wave slip inversions that find that only about the southern 1/3 [*Ji,* 2005] or 2/3 of the area [*Yamanaka,* 2005] slipped. Hence if the body waves and even the 300-s surface waves reflect slip only on the southern 1/3 of the area, the resulting moment is only 1/3 of what the gravest modes record.

It is interesting that, using the radial mode ${}_0S_0$, with period of 1227 s, *Park* [2005] estimates the rupture duration at more than 400 s, consistent with Yamanaka's estimate, but twice as long as Ji's. Hence it seems likely that at least 2/3, and probably all, of the aftershock zone slipped. It is worth noting that the tsunami run-up in the near field on Sumatra is 25-30 m, which implies at least 12-15 m of slip, by a rule of thumb that run-up typically does not exceed twice the fault slip (*G. Plafker,* pers. comm.) that is supported



$_0S_3^{-3}$

MAJO -- Matsushiro, Japan



$_0S_3^{+3}$

Figure 2.: Time series and envelopes for two singlets at MAJO, showing moment and *Q* estimated from least-squares fit.

CONCLUSION of PRELIMINARY STUDY

- The seismic moment of the Sumatra earthquake increases regularly with period (like $T^{1/2}$) from 300 to 3000 seconds.

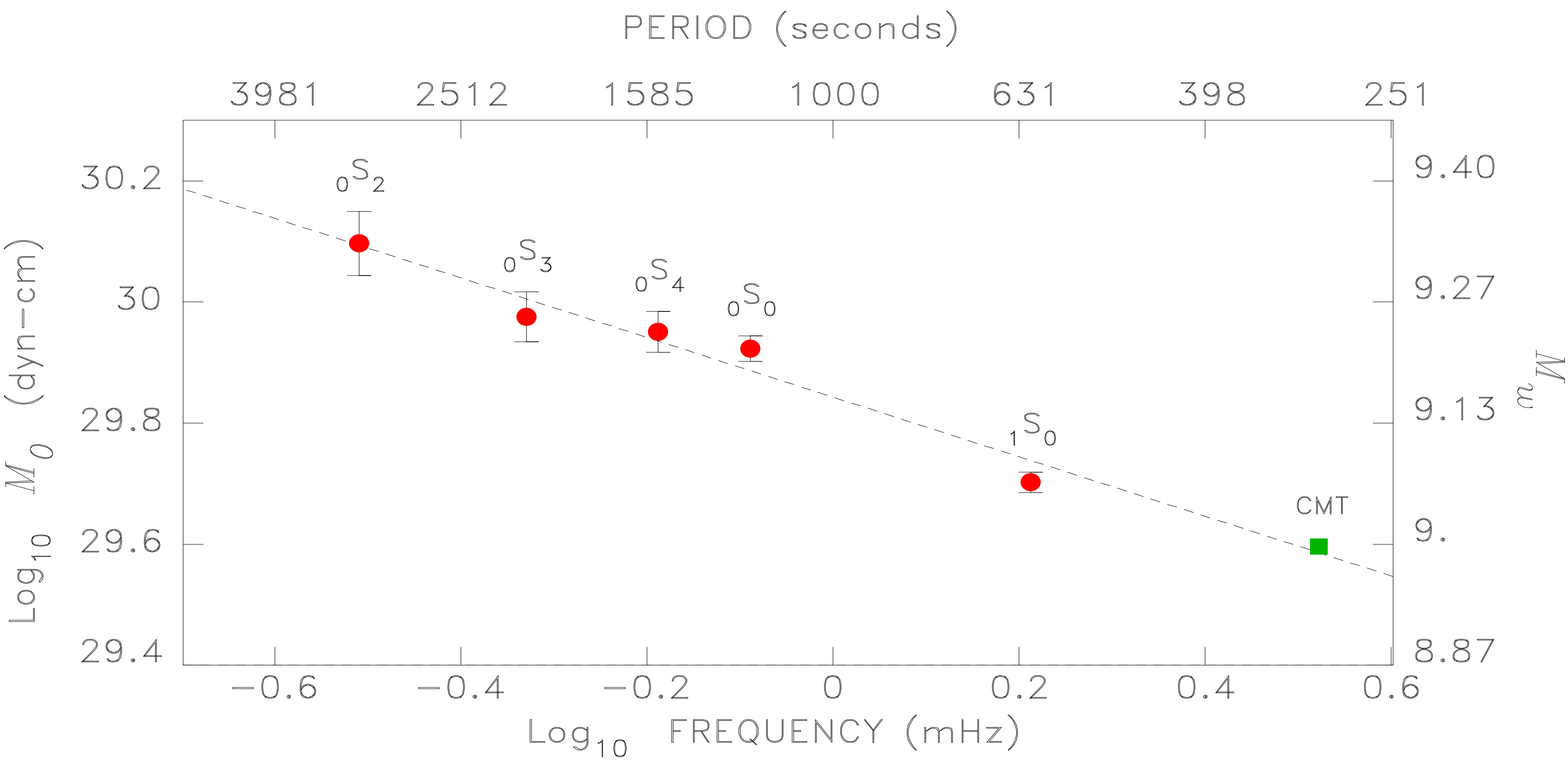


Figure 3.: Variation of moment estimates as a function of period. Note the steady increase at longer periods.

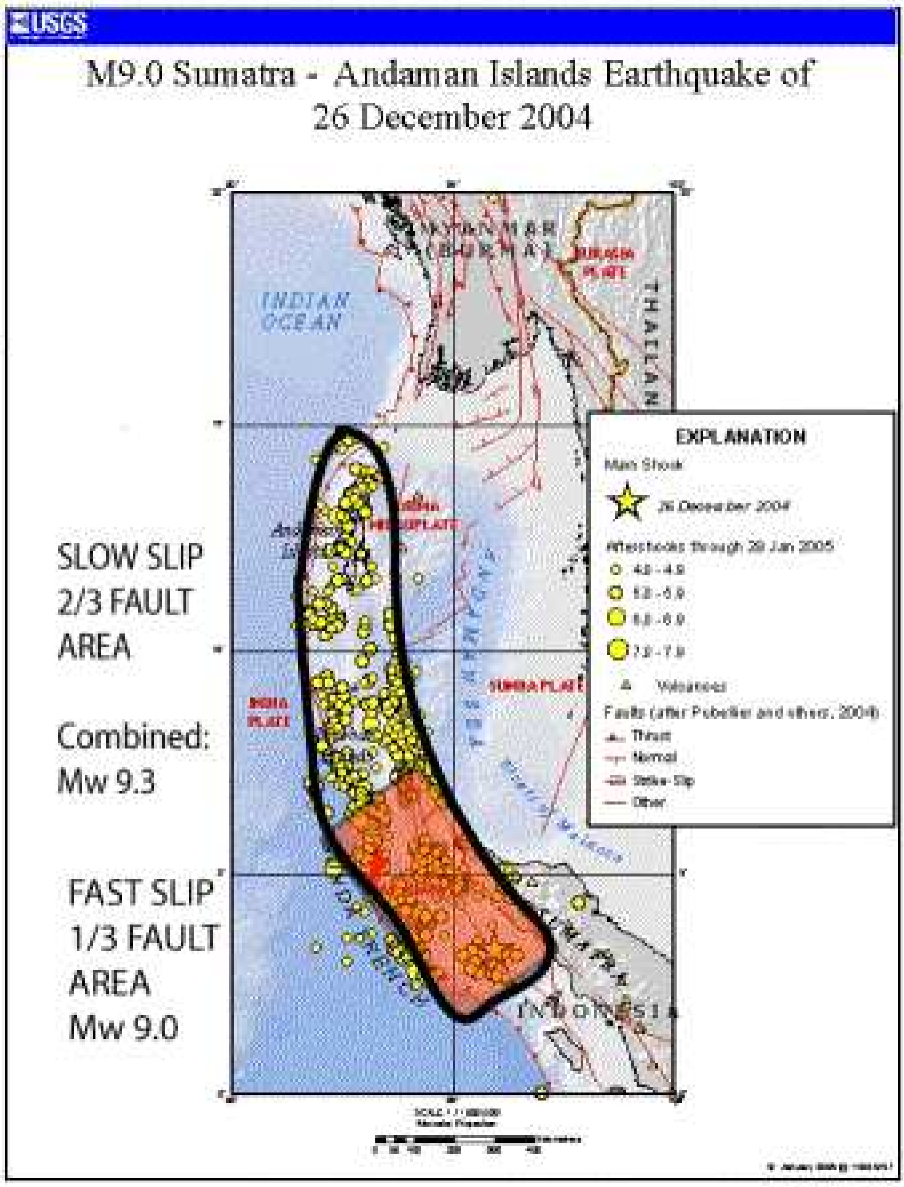


Figure 4.: Schematic illustration comparing aftershock zone to minimum area of fast slip estimated from body waves and possible area of slow slip inferred from normal modes.

Modifi ed from http://neic.usgs.gov/neis/eq_depot/2004/eq_041226 by near-field simulations [*Okal and Synolakis*, 2004].

An interesting question is whether the slow slip contributed to the tsunami excitation. This possibility is suggested by the fact that Titov and Arcas have successfully modeled the amplitude of the tsunami on the high seas, as detected by the JASON satellite, using a source that includes the northern segment.

If the entire aftershock zone slipped significantly, then accumulated strain on the northern 2/3 of the rupture zone has also been released, leaving no immediate danger of a large tsunami being generated by slip on this segment of the plate boundary. However, the danger of a large tsunami resulting from a great earthquake on segments to the south remains.

References

Dahlen, F. A. and R. V. Sailor, Rotational and elliptical splitting of the free oscillations of the earth, *Geophys. J. Roy. astr. Soc.*, **58,** 609−624, 1979.

Geller, R. and S. Stein, Time-domain attenuation measurements for fundamental spheroidal modes (${}_0S_6$ to ${}_0S_{28}$) for the 1977 Indonesian earthquake, *Bull. Seism. Soc. Am.*, **69,** 1671−1691, 1979.

Ji, C., http://neic.usgs.gov/neis/eq_depot/2004/eq_041226/neic_slav_ff.html.

Okal, E. A., and C. E. Synolakis, Source discriminants for near-field tsunamis, *Geophys. J. Intl.*, **158,** 899−912, 2004.

Park., J. http://www.iris.iris.edu/sumatra/

Stein, S. and R. Geller, Amplitudes of the earth's split normal modes, *J. Phys. Earth*, **25,** 117−142, 1977.

Stein, S. and J. Nunn, Analysis of split normal modes for the 1977 Indonesian earthquake, *Bull. Seism. Soc. Am.*, **71,** 1031−1047, 1981.

Tanimoto, T., Lateral variation of Q from singlet modal Q measurements of ${}_0S_2$, *Geophys. Res. Letts.*, **17,** 669−672, 1990.

Yamamaka, Y., http://www.eri.utokyo.ac.jp/sanchu/Seismo_Note/2004/EIC161ea.html