Geol 202 Answers for Problem Set 1

Due: January 15, 2007

If you don't understand something on the problem set answers or if you find an error, e-mail Laura

1. To determine the size of the moon, use the fact that it appears to be the same size as a nickel held 229 cm from the observer (this is called having the same *angular diameters).* If the moon is $3.85\ x\ 10^5\ km$ away, what is the *radius* of the moon? How does the moon compare in volume to the earth?

$$\frac{1.0605cm}{229cm} = \frac{R_{moon}}{3.85\ x\ 10^5 km}$$

$$R_{moon} = \left(3.85\ x\ 10^5 km\right)\ x\ \frac{1.0605}{229} = 1782.9km$$

$$V_{moon} = 4/3\pi R^3 = 4/3\pi(1782.9km)^3 = 2.374\ x\ 10^{10} km^3$$

$$V_{earth} = 4/3\pi(6371km)^3 = 1.08\ x\ 10^{12} km^3$$

The moon's volume is much smaller than the earth's volume.

2. The acceleration of gravity on the lunar surface is about 1/6 that on the earth's surface. Given the moon's radius from problem 1, find its mass and average mean density.

Using Newton's second law and the universal law of gravity we find:

$$F = mg = \frac{GMm}{R^2}$$

We divide each side by m to calculate g.

$$g = \frac{GM}{R^2}$$

Then the variables are rearranged to calculate the mass.

$$M = \frac{gR^2}{G}$$

To find the mass of the moon, plug $g_{moon} = 1/6\ x\ 9.8m/s^2$, $R_{moon} = 1782.9km$, and $G = 6.672\ x\ 10^{-11} Nm/kg^2$ into the equation above, but remember to convert the units first.

$$M_{moon} = \frac{g_{moon} R^2_{moon}}{G} = 7.78\ x\ 10^{22} kg$$

to find the average density of the moon, we divide the mass by the volume.

$$Density_{moon} = \frac{Mass_{moon}}{Volume_{moon}} = \frac{7.78\ x\ 10^{22} kg}{2.374\ x\ 10^{10} km^3} = 3280kg/m^3 = 3.280g/cm^3$$

3. Before actually working problem 2, you might be tempted to say that if the acceleration of gravity on the moon was 1/6 that on earth, its mean density is 1/6 that of earth. Why isn't this correct?

Calculating the average density of a planet uses both planetary mass and volume. Concluding that the moon's mean density is 1/6 that of the earth's density assumes that the volume of the earth and the moon are the same and only the masses differ. However, both the mass and the volume of the moon are different than the earth. Therefore the average density of the moon is not 1/6th the average density of the earth.

4. A model often used for the moon is that it is made of (green) cheese. Test this model by comparing its density to a block of Monterey Jack with dimensions 3 cm x 12 cm x 7 cm and a mass of 255 grams.

The density is the mass divided by volume. The mass is given and the volume is the width, length, and height multiplied together.

$$Density = \frac{Mass}{Volume}$$

$$Density_{MontereyJack} = \frac{255g}{3cm\ x\ 12cm\ x\ 7cm} = 1.01\, g/cm^3$$

The density of the Monterey Jack is much less than the density of the moon. Thus it is unlikely that the moon is made of Monterey Jack cheese.

5. We saw in class that a body in earth's orbit has an orbital period which depends on its distance from the center of the earth. Communications satellites are often put in *synchronous orbit* - that is, with a period equal to one day so that they stay above the same point on the earth as the earth rotates. How large is the radius of this orbit? How high above the earth's *surface* is the satellite?

Using the universal law of gravity and the centripetal acceleration, we calculate:

$$F = \frac{GMm}{R^2} = \frac{mv^2}{R}$$

The variables are rearranged to calculate the mass.

$$M = \frac{Rv^2}{G}$$

The time is takes for the satellite to complete one orbit is:

$$T = \frac{2\pi R}{v}$$

The above equation is rearranged to calculate the velocity.

$$v = \frac{2\pi R}{T}$$

The velocity is plugged into the equation for the mass.

$$M = \frac{R}{G} x \left(\frac{2\pi R}{T} \right)^2 = \frac{4\pi^2 R^3}{GT^2}$$

The above equation is rearranged to find R, the orbital radius.

$$R = \left(\frac{MGT^2}{4\pi^2} \right)^{1/3}$$

Plug the mass of the earth, the universal gravitational constant and T = 86400 s (the number of seconds in one day) into the equation above to calculate:

$$R = 4.2254 \ x \ 10^7 m = 42254 km.$$

The distance from the surface of the earth to the statellite is:

$$Distance = 42254 km - 6371 km = 35883 km$$

8. Moment of inertia factors I/MR^2 (and other information) for the various planets can be found on the website *http://nssdc.gsfc.nasa.gov/planetary/planetfact.html* or on class website *http://www.earth.northwestern.edu/people/seth/202* by clicking on the fact sheet for the planets. Find the values for the Sun, Venus, Mars, the moon, Earth, and Jupiter. Put these in order from large to small and explain what they tell about the density distribution (the way that the density changes from the surface to the center of an object).

The moment of inertia factor ordered from largest to smallest is: Moon, Mars, Earth, Venus, Jupiter, Sun. The moment of inertia factor for the moon suggests that the moon is relatively homogeneous with a very small core. All the planets have smaller moment of inertia factors suggesting they are less homogeneous and the density increases towards the middle of the planet. The sun has the smallest moment of inertia factor suggesting it has the largest density difference between the surface density and the core density.

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planet radius meters	core radius meters	mantle density kg/m^3	core density kg/m^3	a/r	I	M	I/Ma2
6,371,000	2,900,000	7000	7000	0.455	1.23E+38	7.58E+24	0.400
6,371,000	6,307,290	7000	0	0.990	6.03E+36	2.25E+23	0.660
3,386,000	1,500,000	4500	10750	0.443	3.44E+36	8.20E+23	0.365