

Earth 202 Problem Set 5
The Early Solar System

Due Tuesday, February 12

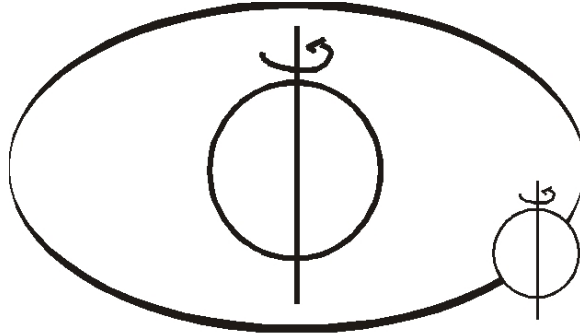
1. (5 points) The gravitational energy binding an object of mass m to a planet of mass M and radius r is $U = GMm/r$. Since this is also the energy that would be released by bringing the object to earth from infinity, this relation is used to calculate the energy in accreting planets.

a) Assuming all this energy is converted to kinetic energy, find the velocity that an object would impact with on earth today.

b) What would the impact velocity be during the early stages of earth formation? Assume the earth had radius 100 km and the present average density.

2. (15 points) Angular momentum in the solar system.

Angular momentum is the momentum associated with rotating bodies. As all bodies in motion have a *linear* momentum, so do all rotating bodies have an associated *angular* momentum. Like linear momentum, angular momentum is conserved, so, in the absence of external torques (or equivalently in linear motion: forces), the angular momentum remains constant with time. In considering the solar system there are two forms of circular motion to consider



a) Rotation of a body about a spin-axis

For a rigid body rotating about a fixed axis, the angular momentum is,

$$L_{rot} = I \omega_{rot},$$

where I is the moment of inertia of the body and ω_{rot} is the angular velocity in radians/sec. Assume that the Sun and planets are rigid bodies (A statement that would make any true celestial mechanic laugh), and recall that the moment of inertia factor for a body of mass M and radius a is equal to;

$$k = I/Ma^2$$

The angular velocity can be calculated by dividing the number of radians (2π radians = 360°) traversed in one rotation by the rotational period, the time it takes for the planet to rotate on its axis. On Earth, for example, this rotation requires 1 day. Using the mass, radius, period and moment of inertia factors given below, calculate the angular velocity, moment of inertia and angular momentum for the bodies listed, **using the units listed in the table!**

Angular Momentum associated with rotation about an axis.							
Body	Rotation Period days	Equatorial Radius km	Mass 10^{24} kg	Moment of Inertia Factor	ω_{rot} rad/sec	Moment of Inertia $\text{kg}\cdot\text{m}^2$	L_{rot} $\text{kg}\cdot\text{m}^2/\text{sec}$
Sun	27	6.96×10^5	2×10^6	0.006			
Earth	1	6,378	5.97	0.331			
Jupiter	0.40	71,900	1900	0.25			
Saturn	0.43	60,200	570	0.22			

To see where most of the rotational angular momentum resides, compare the angular momentum ratio of the Earth/Sun and Jupiter/Sun (Hint: setting up ratios may be the easiest way to do this).

b) Orbital motion of a body about the center of mass.

Assume that the planets represent point masses which revolve about the center of mass of the solar system (which to a very good approximation is the center of the Sun). Also assume that the planets orbit in a plane (known as the *ecliptic*) perpendicular to the spin axis of the sun. You can then approximate the angular momentum associated with the revolution of the planets about the sun by the expression,

$$L_{orb} = mr^2 \omega_{orb}$$

where m is the mass of the planet, r is the orbital radius and ω_{orb} is the angular velocity of the planet in its orbit (which again is 2π radians divided by the time required for a complete orbit). Using the values for orbital period and radius given below, and the masses given in part (a), calculate the angular velocity and angular momentum associated with the orbits of the planets. Enter these in the table below (again in the **units listed**).

Angular Momentum associated with revolution about a point.				
Body	Revolution Period yrs	Mean Orbital Radius 10^6 km	ω_{orb} rad/sec	L_{orb} $\text{kg}\cdot\text{m}^2/\text{s}$
Earth	1	150		
Jupiter	11.9	778		
Saturn	29.5	1427		

Where does most of the orbital angular momentum reside?

c) Sum up all the angular momentum (rotational and orbital) in the solar system (just consider the bodies for which you have calculated the angular momenta). Compare this to the total angular momentum individually associated with each: the Sun, Earth, Jupiter and Saturn. Where does the majority of the angular momentum reside?

3. (14 points) Collisions can be understood with simple considerations of the conservation of momentum, and energy. Two cases are to be illustrated, those involving *elastic* collisions (i.e. the kinetic energy of the colliding bodies is conserved) and *inelastic* collisions (i.e. some or all of the kinetic energy of the colliding bodies has been converted to other forms, e.g. heat). In either case, linear momentum is conserved until the system is acted on by an external source (in accordance with Newton's first law of motion). Thus by vector addition,

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_3 + m_2\mathbf{v}_4 \quad \text{elastic collisions}$$

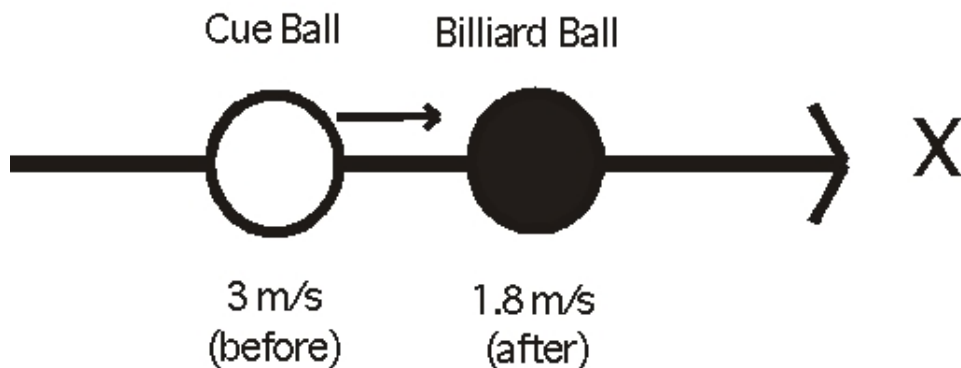
$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}_{12} \quad \text{inelastic collisions}$$

where m_1 and m_2 are the mass of body one and two respectively, \mathbf{v}_1 and \mathbf{v}_2 are the pre-impact velocities of body one and two, \mathbf{v}_3 and \mathbf{v}_4 are the post-impact velocities of body one and two, and \mathbf{v}_{12} is the post-impact velocity of the combined body formed by the inelastic impact of bodies one and two.

We will use billiard balls to illustrate the elastic collisions (e.g. they deform negligibly under collision), planets, however, do not behave elastically but may be closely approximated as undergoing totally inelastic collisions (e.g. the two colliding bodies become one).

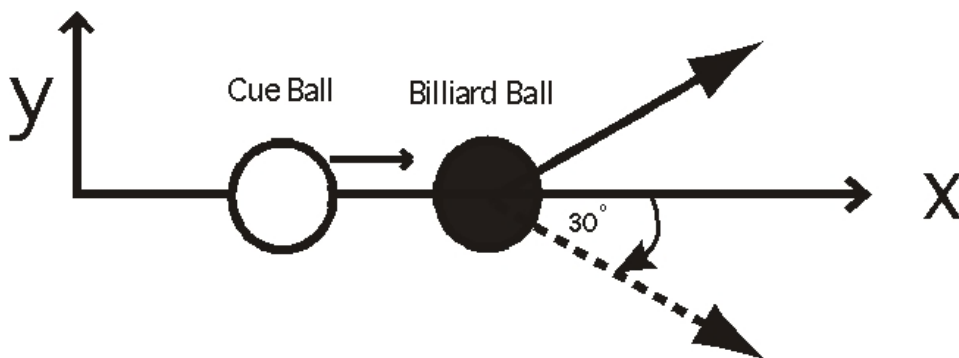
a) *Elastic Collisions*

A cue ball traveling with a **speed** (the magnitude of the velocity) of 3 m/s collides with a stationary billiard ball and imparts a speed of 1.8 m/s to the billiard ball. If the billiard ball is sent forward in the same line as the incident cue ball, what will be the speed of the cue ball after the collision? (Hint: Assume both balls have the same mass and conserve the system's momentum).



b) *Velocity and Elastic Collisions*

Now assume that instead of a straight shot, you wish to send the billiard ball 30° to the side. This requires that you consider the magnitude *and* direction of the motion (termed **velocity**). Given the same two balls, and the initial cue ball velocity of 3 m/s along the x-direction (as shown below), and a velocity for the billiard ball of 1.8 m/s 30° off the x-direction (as shown). Determine the velocity (magnitude *and* direction) of the cue ball after collision (Hint: Conserve momentum independently in both the x- and y-direction).



c) *Inelastic Collisions*

Consider the collision of two bodies of Mass M_1 and M_2 that stick together upon collision. Let M_2 be at rest initially (as if you were standing on the second body watching the first), and let v_1 be the velocity of M_1 before the collision.

- i) Describe the motion of the system after the collision (give the magnitude and direction of the velocity after the collision).
- ii) What is the ratio of the final kinetic energy to the initial kinetic energy (Hint: Use the results from Part (i), you should end up with a function which is only dependent on the masses involved).
- iii) What is the numerical value of this ratio for a planetesimal accreting into a planet (or for that matter for a meteorite striking the Earth)? Explain why this is and give a plausible explanation as to what has happened to it.

4. (3 points) To learn about the The Near Earth Asteroid Rendezvous mission, go to the Geology 202 web page <http://www.earth.northwestern.edu/people/seth/202> and to the link for the Near Earth Asteroid Rendezvous (NEAR) mission under the topic *Meteorites, formation of the planets*. Read the information about this mission, and then the material about the asteroid Eros, NEAR's primary target. Explain briefly what NEAR found about Eros.

Next, go back to the course page, and to the link *Information on the NEAR Mathilde Flyby* and read the material, starting with the press release (you can also watch an animation of the Mathilde Flyby on the mainpage). Based on this, explain the goals of the NEAR Mathilde fly-by. What are some of the initial results?