

## PLATE KINEMATICS

The relative motion between any two plates can be described as a rotation about an *Euler pole*

At any point  $\mathbf{r}$  along the boundary between plate  $i$  and plate  $j$ , with latitude  $\lambda$  and longitude  $\mu$ , the *linear velocity* of plate  $j$  with respect to plate  $i$  is

$$\mathbf{v}_{ji} = \boldsymbol{\omega}_{ji} \times \mathbf{r}. \quad (1)$$

$\mathbf{r}$  is the position vector to the point on the boundary, and  $\boldsymbol{\omega}_{ji}$  is the angular velocity vector or *Euler vector*. Both vectors are defined from an origin at the center of the earth.

The direction of relative motion at any point on the boundary is a small circle, a parallel of latitude *about the Euler pole* (not a geographic parallel about the North Pole!).

Segments of the boundary where relative motion is parallel to the boundary are *transform faults*. Thus transforms are small circles about the pole and earthquakes occurring on them should have pure strike-slip mechanisms.

Segments with relative motion away from the boundary are *spreading centers*.

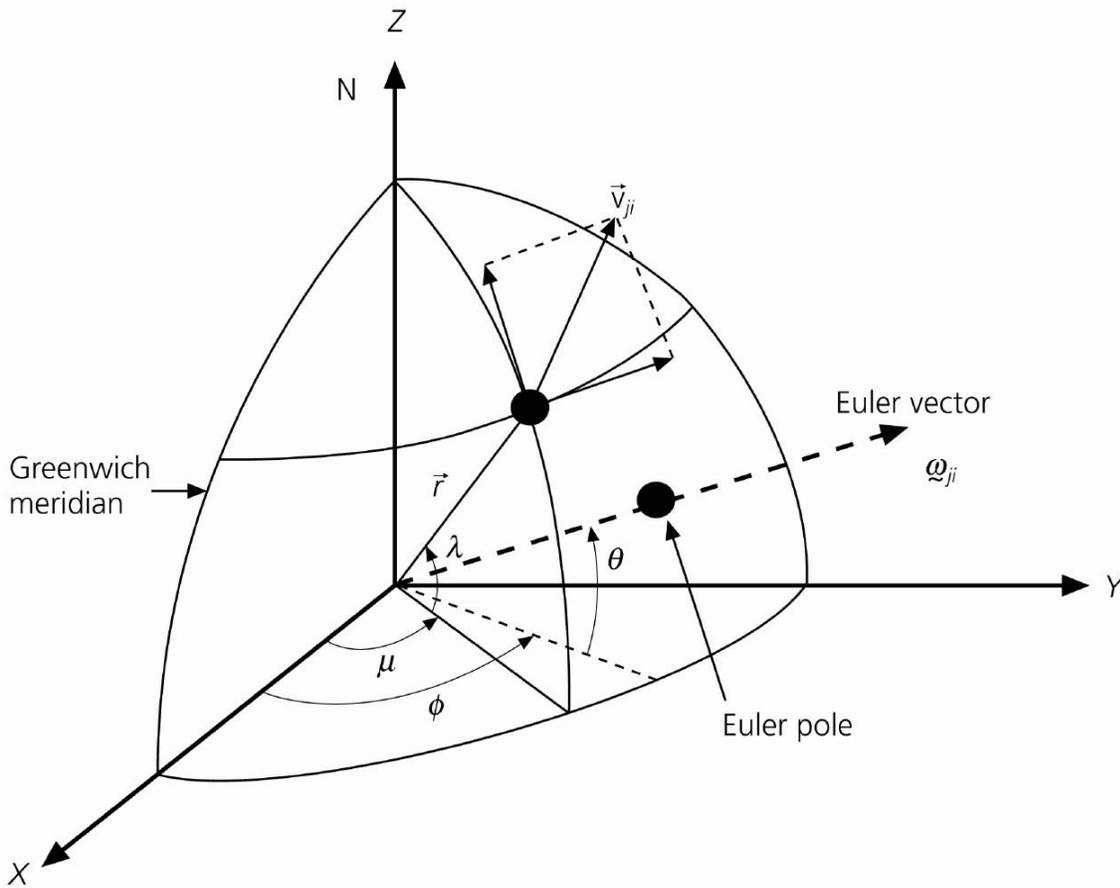
Segments with relative motion toward from the boundary are *subduction zones*.

The magnitude, or rate, of relative motion increases with distance from the pole because

$$|\mathbf{v}_{ji}| = |\boldsymbol{\omega}_{ji}| |\mathbf{r}| \sin \gamma, \quad (2)$$

where  $\gamma$  is the angle between the Euler pole and the site (corresponding to a colatitude about the pole).

All points on a plate boundary have the same angular velocity, but the magnitude of the linear velocity varies from zero at the pole to a maximum  $90^\circ$  away.



In Cartesian  $(x, y, z)$  coordinates (Figure 1) the position vector is

$$\mathbf{r} = (a \cos \lambda \cos \mu, a \cos \lambda \sin \mu, a \sin \lambda), \quad (3)$$

where  $a$  is the earth's radius. Similarly, if the Euler pole is at latitude  $\theta$  and longitude  $\phi$ , the Euler vector is written (neglecting the  $ij$  subscripts for simplicity) as

$$\boldsymbol{\omega} = (|\boldsymbol{\omega}| \cos \theta \cos \phi, |\boldsymbol{\omega}| \cos \theta \sin \phi, |\boldsymbol{\omega}| \sin \theta) \quad (4)$$

where the magnitude,  $|\boldsymbol{\omega}|$ , is the scalar angular velocity or rotation rate.

Using the cross product

$$\mathbf{v}_{ji} = \boldsymbol{\omega}_{ji} \times \mathbf{r} \quad (1)$$

gives the N-S and E-W components of  $\mathbf{v}$  the linear velocity  $\mathbf{v}$

$$v^{NS} = a|\boldsymbol{\omega}| \cos \theta \sin(\mu - \phi) \quad (5)$$

$$v^{EW} = a|\boldsymbol{\omega}| [\cos \lambda \sin \theta - \sin \lambda \cos \theta \cos(\mu - \phi)].$$

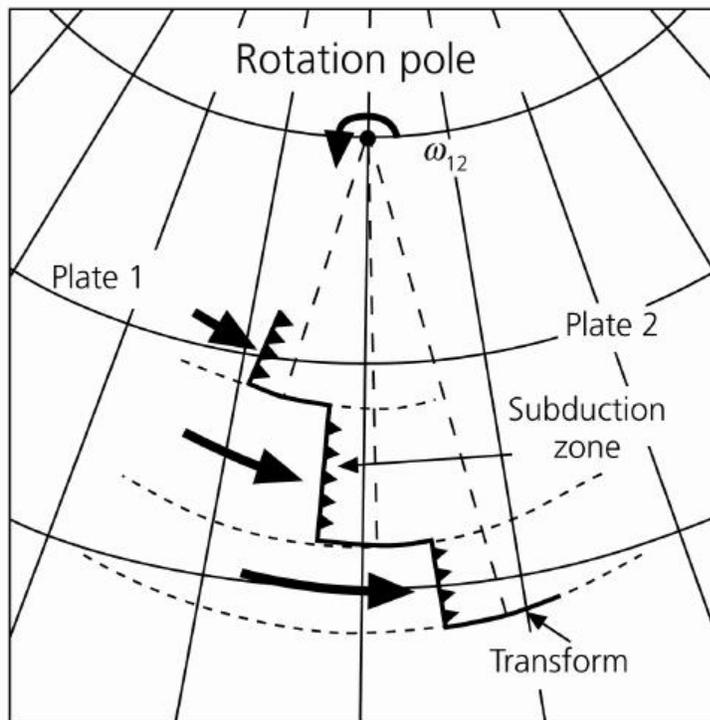
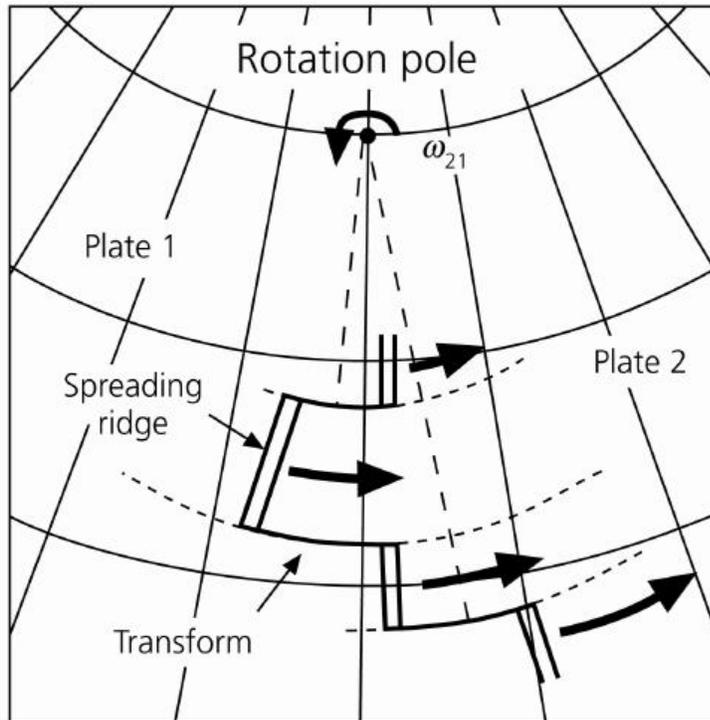
The rate and direction of plate motion are

$$\text{rate} = |\mathbf{v}| = \sqrt{(v^{NS})^2 + (v^{EW})^2} \quad (6)$$

$$\text{azimuth} = 90^\circ - \tan^{-1}[(v^{NS})/(v^{EW})],$$

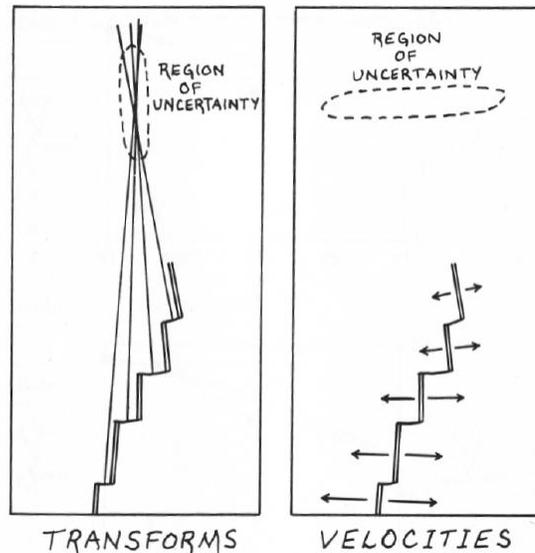
such that azimuth is measured in the usual convention, degrees clockwise from North.

Rotation rates in degrees/Myr should be converted to radians/yr, giving linear velocity with the same dimensions as Earth's radius. Converting radius in km to mm ( $\times 10^6$ ) and Myr to years ( $\times 10^6$ ) cancel out, so only the degrees to radians ( $\times \pi/180^\circ$ ) conversion needs to be done to obtain a linear velocity in mm/yr. Plate motions are often quoted as mm/yr, because 1 mm/yr corresponds to 1 km/Myr, making it easy to visualize what plate motion accomplishes over geologic time.



## FINDING A POLE AND RATE - RIDGE/TRANSFORM EXAMPLE

1) TRANSFORM DIRECTIONS measured from underwater or on-land mapping, or earthquake slip directions. Transforms are small circles (like parallels) so draw meridians (great circles perpendicular to transforms). Great circles meet at the pole, with some scatter, of course. This has good "longitude" (with respect to pole) control, bad "latitude" control



2) SPREADING RATE measured from magnetic anomalies.  $V \sim \sin \alpha$ , where  $\alpha$  is the "latitude" from pole, so the variation constrains the pole location. This has good "latitude" control, bad "longitude" control

At subduction zone - no good velocity information but can get direction from earthquake slip vectors .

## MORE ADVANCED METHOD

Use analytic expression to find best-fitting pole for all data points. Determination of Euler vectors for all plates is an overdetermined least squares problem whose solution gives a *global relative plate motion model*. These models use spreading rates determined from magnetic anomalies that span several Myr. The most recent magnetic reversal occurred about 780,000 years ago, so a *plate model based on paleomagnetic data averages motion at least over that interval*.

## VECTOR ADDITION

Given Euler vectors with respect to one plate those with respect to other plates found by vector arithmetic.

For example,

$$\omega_{ij} = -\omega_{ji}, \quad (7)$$

so we reverse the plate pair using the negative of the Euler vector. The pole becomes the antipole, with latitude of opposite sign and longitude increased by  $180^\circ$ . The magnitude (rotation rate) stays the same. We can also reverse the plate pair by keeping the same pole and making the rotation rate negative (CW rather than CCW).

For other plate pairs, we assume that the plates are rigid, so all motion occurs at their boundaries. We then add Euler vectors,

$$\omega_{jk} = \omega_{ji} + \omega_{ik} \quad (8)$$

because the motion of plate  $j$  with respect to plate  $k$  equals the sum of the motion of plate  $j$  with respect to plate  $i$  and the motion of plate  $i$  with respect to plate  $k$ . Thus starting with a set of vectors with respect to plate  $i$ , we use

$$\omega_{jk} = \omega_{ji} - \omega_{ki} \quad (9)$$

to form any Euler vector. These operations are done using the Cartesian components (4). We can also perform these vector additions on linear velocity vectors at a specific point.

*Vector addition is important because we only have certain types of data for individual boundaries.* Although spreading centers provide rates from the magnetics and azimuths from transforms and slip vectors, only the direction of motion is directly known at subduction zones. Hence convergence rates at subduction zones are estimated by *global closure*, combining data from all plate boundaries.

## HOTSPOTS

Everything we've done is in terms of relative motions between plates. Can we go beyond this to an *absolute* motions...with respect to WHAT?

Curious thing - look at globe, see some wierd thing that aren't ridge's classic example: Hawaii, turns into chains of seamounts (volcanoes). The "hot spot" idea is there are magma source below lithosphere *which are fixed - unlike plates*

See others (Atlantic - hot spot under ridge)

To test this idea - use K/Ar dates (MyBP) (DSDP samples on seamounts). It works! If this idea is right - and one can find about 20 such hotspots - we can define an *absolute reference frame* for each plate.

$$V_j = \Omega_j \times r, \quad \Omega_j \text{ (absolute wrt hotspots)}$$

so the relative motions can be proposed

$$\omega_{ji} = \Omega_r - \Omega_i$$

(j, i need not have common boundary) so as, we claimed

$$\omega_{ij} = \Omega_i - \Omega_j = -\omega_{ji}$$

To get absolute motions, form all relative velocities wrt Pacific  $\omega_{jp}$  where  $p$  is Pacific. Then form, for example,  $\omega_{pH}$  Pacific wrt hotspots. So get

$$\omega_{jp} + \omega_{pH} = \omega_{jH} = \Omega_j$$

Other possible tool: equatorial sediment buldge (upwelling, CCD down, get organic ooze)

The nature of hotspots is a very important and active research area (Geol. 325). Other hotspots: Yellowstone, Waluis/Rio Grand, etc...

Important - Hawaii's bend indicates plate motion change.

Mantle plumes from CMB?

## HOW DOES PLATE TECTONICS WORK?

We know what happens - why does it occur? What causes plate motions?

Since most of the action is taking place at the ridges and the trenches, lets see what's up... We don't fully understanding these - its a major research area - but there are some ideas... Evolution of the lithosphere:

What happens to the seafloor once its formed at a ridge?

Measurement of heat flow

$$q = -k \frac{dT}{dz} \text{ at } z = 0$$

1) measure  $dT/dZ$

2) find  $k$

a) brings up core

b) in-situ (heat up the sample)

*conceptual* - real instruments are more complicated

curiously, topography (bathymetry) does the same thing. Why?

## THERMAL EVOLUTION OF OCEANIC LITHOSPHERE

The basic process driving plate tectonics is thermal evolution of lithosphere. Primary observation: with age, sea floor depth increases and heat flow decreases. Why? Lithosphere is formed hot at ridges, then cools and so subsides.

Simple model: consider column of lithosphere formed at ridge temperature  $T_m$ , time 0. To see what happens to the column as a function of time and depth heat equation without radioactivity.

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial z^2} = \kappa \frac{\partial^2 T}{\partial z^2} \quad (\kappa = \frac{k}{\rho C_p} \text{ is called thermal diffusivity})$$

A solution (we won't prove this!) is

$$T(z, t) = T_m \operatorname{erf} \left[ \frac{z}{2\sqrt{\kappa t}} \right]$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is called the error function and is tabulated in math books.

What does this tell? At a depth  $Z$ , the temperature reaches most of its final value when

$$\frac{z}{2\sqrt{\kappa t}} = x = 1, \text{ or } z = 2\sqrt{\kappa t}$$

In other words, the depth reached goes as the square root of the time. In fact, any *isotherm*, a line of constant temperature, is just constant  $= \frac{Z}{2\sqrt{\kappa t}}$ , or  $Z = \text{constant } 2\sqrt{\kappa t}$

So if the lithosphere is defined as depth to an isotherm ( $\sim 1100^\circ \text{C}$ ) its thickness increases with (age)<sup>1/2</sup>. We can solve mathematically the heat equation problem in which a ridge is at temperature  $T = T_m$   $1400^\circ \text{C}$  and moves away at velocity  $V$ .

Tests:

1) Below A depth where temperature  $\sim 1000^\circ \text{C}$ , the seismic wave velocity drops

2) Heat flow

$$q = k \frac{dT}{dz} = k \frac{\Delta T}{\Delta z} = k \frac{T_c}{z_c} = \frac{k T_c}{ct^{1/2}}$$

So heat flow decreases with (age)<sup>-1/2</sup>. Topography also changes with age. Consider mass in two columns to depth  $L$

$$\rho_l(L) + \rho_a(L - l) = \rho_w h + \rho_l z_c(t) + \rho_a(L - z_c(t) - h(t))$$

$$h(t) = (z_m(t) - l) \frac{(-\rho_l + \rho_a)}{(\rho_w - \rho_a)}$$

This mass balance idea is called isostasy. So, since  $z_m(t) \sim \sqrt{t}$ ,  $h(t) \sim \sqrt{t}$  (can use to get  $\rho_l - \rho_a$  contrast. Oceans deepen as  $\sqrt{t}$ . In reality, its slightly more complicated because the depth layer curve flattens at about 70 Ma - it no longer deepens as  $\sqrt{t}$

What is thought to be happening is that the rate at which heat is lost from the top by cooling is balanced by heat from below, depth  $a$ , so the temperature structure equilibrates.

$$T(z) = T_0 + \frac{(T_m - T_0)}{az}$$

the equilibrium heat flow is

$$q = \frac{dT}{dz} = 0 = \kappa(T_m - T_0) = \frac{(T_m - T_0)}{a}$$

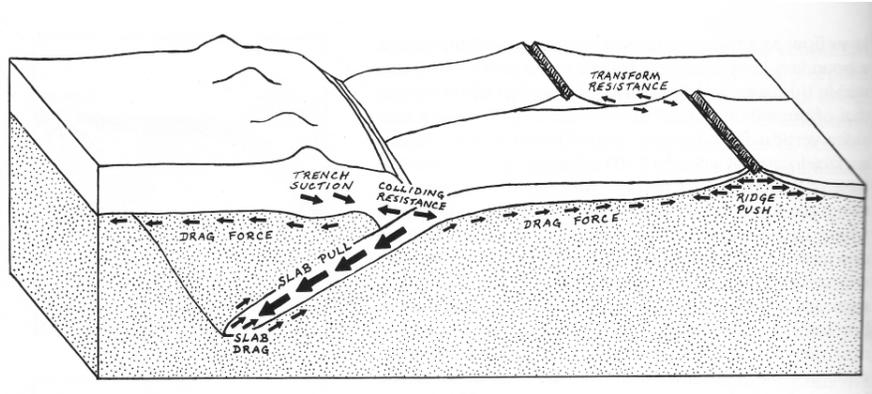
Thus lithosphere has equilibrium thickness

$$a = \frac{\kappa}{q} (T_m - T_0)$$

So, the simple thermal model explains

- 1) topography
- 2) heatflow

It also gives one of the major forces driving the plates - *ridge push*.



$\rho_1 > \rho_a$  get "downhill sliding" ridge push. Consider the net force

$$F_1 - (F_2 + F_3) = F_R \text{ on lithosphere}$$

$$F_R(t) = g\alpha\rho_m T_m \kappa t$$

in spreading direction.  $F_1$  - pressure of asthenosphere at ridge and  $F_2$  - pressure of water at ridge.

At the other end are *subduction zones*. Sometimes spreading occurs behind the arc.

Earthquakes go down to 600 km - define the Benioff Zone, show where plate is going down. The largest earthquakes in the world occur here - related to subduction process. How can earthquakes occur at this depth? Usually, as go deeper get hotter softer stuff - so material is too weak. But here: - (cold interior NVE 165)

As the slab goes down - it is much colder than the surrounding material - it slowly heats up eventually equilibrates. This gives a sinking force due to the density difference between the cold (dense) material of the slab and the hot (less dense) surrounding material.

$$F = \int_0^l \int_0^L g [\rho(T) - \rho(T')] dx dz$$

If no subduction temperature here would be  $T$ : with subduction its  $T'$  ( $T' < T_2$ ), but we have the coefficient of thermal expansion,  $\alpha$ , so

$$\rho(T) - \rho(T') = \rho [T - T'] \alpha$$

get

$$\text{Force} = \text{const} \frac{(g\alpha\rho^2 l^3 v)}{2k}$$

makes sense - if conductivity ( $k$ ) is high, slab can equilibrate faster, reduces force. Large velocity ( $v$ ), thickness ( $l$ ): slab takes longer to equilibrate - increases force. This force is called "slab pull or trench pull"

Both ridge push and slab pull are "thermal buoyancy forces" ie due to density differences resulting from temperature

structure. Test if this is important...NVE 199 the more slab, the faster a plate moves... Driving force are a major research area.

Final idea: back arc basins. As slab sinks - it "sucks" down asthenosphere which is replaced giving "back-arc basins". Lots of work going on to understand how.