VIII. THERMAL EVOLUTION OF THE PLANETS

8.1 HEAT TRANSFER

Given that the planet formed, the issue at hand is to understand how it evolved to its present state and how it may still evolve in the future. The primary effect is the evolution of the earth's temperature distribution: its *thermal history*. **Heat is the geological lifeblood of planets**.

To study this, we need to get some basic concepts:

HEAT is a measure of the internal *energy* of a body

TEMPERATURE is a measure of the heat contained and the ability to transfer heat

For example, if we change the temperature of a body by ΔT, the heat change per unit volume is

$$\Delta U = \rho C_p \Delta T$$

where C_p is the specific heat per unit mass, this tells us how much the object will heat up for a given temperature change, and ρ is the density (mass per unit volume).

Heat has the units of energy (joules, ergs, or calories), and C_p has units of energy/degree-mass (joules/°C kg).

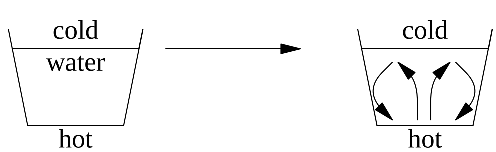
Some unit relationships: erg = (dyne)(cm)
= 10^{-7} joules
calorie = 4.2 joules
joule = N m

A useful physical analogy is Feynman's - heat is like the amount of water in a towel, while temperature is the "wetness." Given two towels of the same "wetness" the bigger one contains more water. A small towel can be much "wetter" but contain less water than a big towel.

Heat can flow from one place to another by several methods:

Conduction - heat is transferred by molecular collisions through a solid object. If you touch something hot, the heat is transferred into your fingers by conduction.

Convection - heat is transferred by motion of the material - for example, by a hot fluid. As the bottom of the pot heats up the water starts to convect, as hot water rises, transporting heat upward. Solid rocks convect at temperatures found in the mantle.



Radiation - heat is transferred by electromagnetic waves, the way the sun heats the earth. This effect isn't that important in solid rock, but is crucial at the planet's surface.

8.2 HEAT CONDUCTION

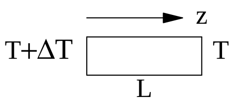
Heat flows from a hot object (one at higher temperature) to a colder one (lower temperature) according to *Fourier's Law of Conduction*

$$q = -k \frac{dT}{dz}$$

where q is heat flux, flow of heat *per unit time* and *unit area*,

and k is thermal conductivity. Materials with k large conduct well - low k materials insulate.

Example:



$$\frac{dt}{dz} = \text{temperature gradient}$$

$$= \frac{T - (T + \Delta T)}{L} = -\frac{\Delta T}{L}$$

$$q = k\,\frac{\Delta T}{L}$$

Notice that the minus sign is set up to make heat go the right way - from hot to cold. (Imagine the consequences if the minus sign weren't there - warm objects would spontaneously heat up!)

In the 1800s it was recognized that temperature in mines and caves increased with depth:

$$\text{typically } \frac{dT}{dz} = \sim 25\ ^\circ\text{C/km}$$

conductivities were measured to show k ~2 W/(m °C) $\longrightarrow$ (Watt=Joule/s) so, q = ~50 mW/m^2 or (in other words) ~1 x $10^{-6}\ \frac{\text{cal}}{\text{cm}^2\ \text{s}}$

you may see this written in "heat-flow units" (HFU)

1 HFU = 10^{-6} cal/(cm^2 s)

= 42 $\frac{\text{mW}}{\text{m}^2}$

(Note that these are per unit time)

On continents, heat flow is typically ~60 mW/m^2 and rock thermal conductivities are ~3 W m^{-1}K^{-1} so,

$$\frac{dT}{dz} = \frac{q}{k} = \frac{6 \text{ x } 10^{-2}}{3} = 2 \text{ x } 10^{-2}\ \frac{K}{m}$$

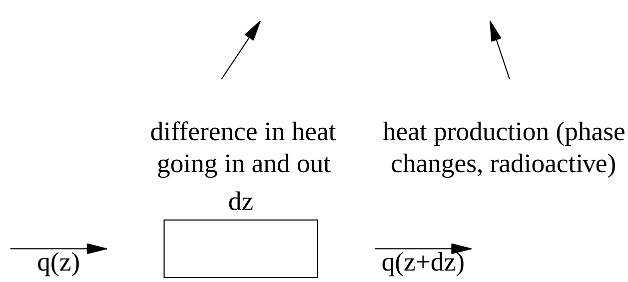
= 20 K/km is the temperature gradient.

8.3 THE HEAT EQUATION

How does the heat, and hence temperature, inside an object change with time? Assume only two methods of change: conduction and production

HEAT CHANGE INSIDE = HEAT CONDUCTED IN OR OUT + HEAT PRODUCED

$$\rho\, C_p\, dz\, \frac{\partial T}{\partial t}(z, t) = q(z) - q(z + dz) + \rho\, H\, dz$$



The Conduction Term

$$q(z) - q(z+dz) = q(z) - \left(q(z) + \frac{dq}{dz}\,dz \right) = -\frac{dq}{dz}\,dz$$

so - we have temperature as a function of position (z) and time (t) in T(z,t)

Using Fourier's law of heat conduction

$$q = -k\,\frac{\partial T}{\partial z}$$

$$\frac{dq}{dz}\,dz = \left(-k\,\frac{\partial^2 T}{\partial z^2} \right) dz \quad \text{(if k is constant)}$$

$$= -k\,dz\,\frac{\partial^2 T}{\partial z^2}$$

ASIDE $\longrightarrow$ Heat produced inside, assume H = heat production rate per unit mass, then

$$\rho H dz$$

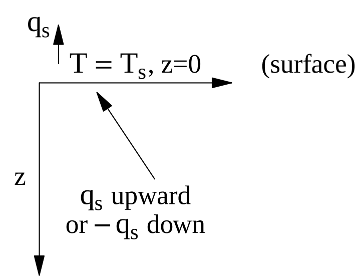
(we'll discuss later how heat is produced in the Earth, but here's an answer: radioactive)
Equating our equations (dividing by dz) with our new conduction term gives

$$\rho C_p\,\frac{\partial T(z,t)}{\partial t} = k\,\frac{\partial^2 T(z,t)}{\partial z^2} + \rho H$$

This animal is called the 1-dimensional (only z coordinate) heat equation with conduction (we didn't worry about convection) IT IS VERY IMPORTANT! Since it tells how the temperature changes with time and position.

8.4 GEOTHERMS

Let's use the heat equation to find temperature in the earth-assuming that the temperature isn't changing with time. (this is called a *steady state* ($\frac{\partial T}{\partial t} = 0$) solution).
We know the heat flow at the earth's surface $-q_s$, and the temperature at the surface T_s.



The heat equation is then

$$0 = k\,\frac{\partial^2 T}{\partial z^2} + \rho H$$

we then integrate with respect to z

$$-k\,\frac{\partial T}{\partial z} + C_1 = \rho H z$$

C_1 is a constant of integration - to find it consider z = 0, and remember that the heat flow at the surface is

$$-q_s = -k\frac{\partial T}{\partial z} \quad \text{at} \quad z = 0$$

so, $C_1 = q_s$

Now we have

$$q_s - k\frac{\partial T}{\partial z} = \rho H z$$

integrate again with respect to z

$$q_s z - kT + C_2 = \frac{\rho H z^2}{2}$$

C_2 is a second constant - to find it recall that at z = 0, T = T_s, so $C_2 = kT_s$

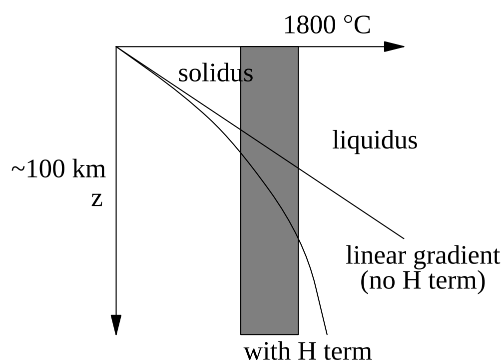
thus: we get the "conduction geotherm equation"

$$T(z) = T_s + \frac{q_s}{k}z - \frac{\rho H}{2k}z^2$$

Assume

- $q_s = 70\ mW/m^2$
- $\rho = 3\ g/cm^3$
- $H = 10^{-10}\ W/kg$
- $K = 4\ W/(m°C)$

We find approximately a curve for T(z) - a *geotherm*



Another line is plotted at the solidus, where rock begins to melt, and at the liquidus, where all the rock has melted.

According to this model, the temperatures at only a few hundred km deep are so high that all the rocks would have melted. This CAN'T be right! We know that shear waves go through the solid upper mantle. Our equation for a geotherm without any heat "sources" (similar to Kelvin's calculation of the age of the earth) would melt the mantle at approximately 100km! Adding heat sources improves the situation, but not much!

The explanation for this erroneous conclusion is that when rock gets hot it convects, it behaves somewhat like a liquid. The complete explanation of how convection works is further than we want to go - instead we'll decide that convection has to be very important in the earth and try to understand some basic ideas.