Mechanisms of anomalous compressibility of vitreous silica

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The anomalous compressibility of vitreous silica has been known for nearly a century, but the mechanisms responsible for it remain poorly understood. Using GHz-ultrasonic interferometry, we measured longitudinal and transverse acoustic wave travel times at pressures up to 5 GPa in vitreous silica with fictive temperatures \(T_f\) ranging between 985 °C and 1500 °C. The maximum in ultrasonic wave travel times—corresponding to a minimum in acoustic velocities—shifts to higher pressure with increasing \(T_f\) for both acoustic waves, with complete reversibility below 5 GPa. These relationships reflect polymorphism in the supercooled liquid, which results in a glassy state possessing different proportions of domains of high- and low-density amorphous phases (HDA and LDA, respectively). The relative proportion of HDA and LDA is set at \(T_f\) and remains fixed on compression below the permanent densification pressure. The bulk material exhibits compression behavior systematically dependent on synthesis conditions that arise from the presence of floppy modes in a mixture of HDA and LDA domains.

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I. INTRODUCTION
The structure of vitreous silica (\(\nu\)-SiO\(_2\)) consists of a fully polymerized, continuous random network of corner-sharing SiO\(_4\) tetrahedra that becomes softer, rather than stiffer, when compressed. The anomalous compressibility behavior persists on hydrostatic loading up to 2–3 GPa at room temperature [1–4]. Bridgman and subsequent workers demonstrated that the deformation of the network is fully reversible below the onset of permanent densification at ∼8 GPa [2–5]. It is also well known that the longitudinal and transverse acoustic wave velocities for \(\nu\)-SiO\(_2\) decrease with increasing pressure up to ∼2.5 GPa, consistent with the anomalous compressibility found in this pressure range [6–11].

Bridgman [2] posited that \(\nu\)-SiO\(_2\) softens on compression because of its relatively open network structure and the manner of collapse on compression. Permanent densification at high pressures and temperatures is accommodated by changes in ring structures and Si coordination number [12,13]. Below the onset of permanent densification, these structural rearrangements are not active, and the rotation of the SiO\(_4\) tetrahedra is proposed to be the active densification mechanism. However, the origin of anomalous compressibility in \(\nu\)-SiO\(_2\) remains controversial, with contrasting models of tetrahedral rotation advanced in the literature.

Vukcevich [14] and others [15–17] proposed a two-state structural model whereby \(\nu\)-SiO\(_2\) transforms from a low-density amorphous phase to a high-density amorphous phase (LDA and HDA, respectively), i.e., polymorphic phase transition, with increasing pressure or temperature, analogous to the \(\alpha\)-\(\beta\) transition in crystalline silica. The displacive \(\alpha\)-\(\beta\) phase transition in cristobalite is accommodated by the tetrahedral rotation of six-member rings. The \(\beta\) phase (low-density polymorph) has more symmetric intertetrahedral bonding than the \(\alpha\) phase (high-density polymorph) making it stiffer, resulting in a higher bulk modulus (\(K\)). If LDA (\(\beta\)-like) and HDA (\(\alpha\)-like) phases possess analogous properties to the cristobalite polymorphs as proposed by the two-state model, the compressibility of \(\nu\)-SiO\(_2\) should depend on their relative stability and, if both are stable due to spinodal decomposition, then by their relative abundance [16,17]. Below and above the spinodal region, \(\nu\)-SiO\(_2\) should behave normally, governed by \(K\) for LDA and HDA, respectively, whereas within the two-phase region the conversion of LDA domains to HDA domains with increasing pressure will lead to softening of the bulk material. In this model, the pressure interval of anomalous compressibility reflects the limits of the coexistence of LDA and HDA.

Experimental evidence for a spinodal in \(\nu\)-SiO\(_2\) comes from the unusual \(T_p\)-\(p\) relations at low pressure. Dilatometric and spectroscopic data show that density increases anomalously between ∼950 °C and at least up to 1500 °C, whereas density decreases with temperature for liquid SiO\(_2\) above ∼1935 °C [18–21]. These relationships imply an underlying phase transition extending over a finite temperature interval with a negative Clapeyron slope, where the low-temperature LDA phase is stable below the spinodal and the high-temperature HDA phase is stable above it [21].

An alternative model, the flexibility hypothesis advanced by Dove and co-workers [22–24] among others, calls on a purely dynamic response to compression to explain the anomalous compressibility of \(\nu\)-SiO\(_2\) involving low-energy deformation by rotation of rigid SiO\(_4\) tetrahedral units freely hinged to coordinated tetrahedra giving rise to so-called floppy modes. Mode softening results from the weakening of the network as the undeformed SiO\(_4\) tetrahedra dynamically rotate during compression. As the network buckles, the material softens until short-range repulsive forces impede further collapse of structural voids [25]. The network flexibility model accounts for anomalous compressibility; however, it does not readily explain the increase in density with temperatures found on
heating $\nu$-SiO$_2$ at 1 atm. It also emphasizes that polymorphism and floppy modes are not mutually exclusive [15,22], but establishing their relative importance in densification of $\nu$-SiO$_2$ remains a challenging yet fundamental step towards further understanding deformation of amorphous materials composed of open tetrahedral networks.

In this study we examined ultrasonic wave propagation in $\nu$-SiO$_2$ during compression at room temperature for samples synthesized at 1 atm but quenched from different temperatures between the density minimum at $\sim$950°C and maximum at $\sim$1935°C. fictive temperatures ($T_f$) of these samples range from 985°C to 1500°C, corresponding to a density increase of $\sim$0.3%. We predict that if both LDA and HDA domains with different compressibility coexist and their relative abundance is a function of temperature, then glasses with different $T_f$ will return to normal behavior at different pressures. In contrast, if the anomalous compressibility is due to floppy rotational modes in a single phase, then either the compressibility will be independent of $T_f$, or $\nu$-SiO$_2$ with a higher initial density (i.e., higher $T_f$) will reach rotational lockup and return to normal behavior at lower pressures.

II. EXPERIMENTAL METHODS

Three high-purity $\nu$-SiO$_2$ samples with different $T_f$ were synthesized at Corning Inc. The samples were prepared by chemical vapor deposition and annealed at 985°C, 1300°C, and 1550°C for 1500, 4, and 0.5 hours, respectively, followed by quenching in air [21]. Fourier transform infrared (FTIR) spectra were collected using a Bruker TENSOR 37 FTIR spectrograph and HYPERION microscope with high-sensitivity MCT detector. We used a 15-mm pixel size detector in conjunction with 15× all-reflecting IR objective. Data were collected at 1 cm$^{-1}$ resolution and 256 scans. $T_f$ of these samples was determined from the position of the $\sim$2260 cm$^{-1}$ overtone band in the infrared absorption spectra with $T_f$ $\sim$985°C, 1260°C, and 1500°C, respectively, for the three samples listed above [see (26), and Supplemental Material [27]]. The densities of the samples at ambient pressure were determined by Archimedes’ method and are reported together with other material properties in Table I.

We used GHz-ultrasonic interferometry in a diamond anvil cell to measure the travel times of acoustic longitudinal ($P$) and transverse ($S$) wave tone bursts through the three $\nu$-SiO$_2$ samples as a function of hydrostatic pressure (Fig. 1). The 1-atm $P$- and $S$-wave velocities for all three $\nu$-SiO$_2$ samples were determined using $\sim$1-mm double-polished plates. Samples for high-pressure measurements were 150-$\mu$m-diameter disks cored from a double-polished thin section $\sim$50 $\mu$m thick. Gigahertz $P$-wave transducers consisted of ZnO thin films ($\sim$1.5 $\mu$m thickness) sputtered onto single-crystal sapphire buffer rods. The buffer rods are mechanically coupled to the polished-plate sample for the 1-atm measurements and to the diamond anvil in contact with the sample for the high-pressure measurements. For $S$-wave buffer rods we used a single-crystal yttrium aluminum garnet (YAG) prism wherein a $P$- to $S$-wave conversion takes place before entering the sample for 1-atm measurements or the diamond anvil for high-pressure measurements. We first measured the travel time of the high-pressure samples on the cuvet of the open diamond anvil cell to determine the initial travel time ($t_0$). The cell was loaded with a ruby pressure marker [28] and

| TABLE I. Physicochemical characteristics of $\nu$-SiO$_2$ samples used in this study at ambient conditions. |
|---------------|---------------|---------------|
| $\nu$-SiO$_2$ sample A | $\nu$-SiO$_2$ sample B | $\nu$-SiO$_2$ sample C |
| Density (g cm$^{-3}$)$^a$ | 2.199 ± 0.002 | 2.203 ± 0.002 | 2.207 ± 0.002 |
| OH content (ppm)$^b$ | 268 ± 7 | 79 ± 4 | 244 ± 5 |
| $T_f$ (deg C)$^c$ | 985 ± 20 | 1260 ± 30 | 1500 ± 40 |
| $V_p$ (m/s)$^{d,e}$ | 5902 ± 1.0 | 5944 ± 1.0 | 5977 ± 0.9 |
| $V_s$ (m/s)$^{d,e}$ | 3765 ± 0.6 | 3757 ± 0.6 | 3751 ± 0.6 |
| $K$ (GPa)$^{f,g}$ | 35.0 ± 0.15 | 36.4 ± 0.15 | 37.4 ± 0.15 |
| $G$ (GPa)$^{f,g}$ | 31.2 ± 0.04 | 31.1 ± 0.04 | 31.1 ± 0.04 |
| $P$-wave linear fit | y = 0.0737x + 0.0019 | y = 0.0656x + 1.0104 | y = 0.0563x + 1.024 |
| (Positive limb) | $R^2 = 0.99263$ | $R^2 = 0.9887$ | $R^2 = 0.98821$ |
| (Negative limb) | $R^2 = 0.99218$ | $R^2 = 0.99133$ | $R^2 = 0.99355$ |
| Intersection (GPa) | 1.92 | 2.17 | 2.20 |
| $S$-wave linear fit | y = 0.0646x + 1.0003 | y = 0.0571x + 0.9947 | y = 0.0548x + 1.0077 |
| (Positive limb) | $R^2 = 0.99556$ | $R^2 = 0.99349$ | $R^2 = 0.99279$ |
| (Negative limb) | $R^2 = 0.99098$ | $R^2 = 0.98226$ | $R^2 = 0.97513$ |
| Intersection (GPa) | 2.05 | 2.35 | 2.39 |

$^a$Measured by Archimedes’ method.
$^b$Measured by FTIR spectroscopy and calculated by the method of [32]. See Appendix A in the Supplemental Material [27].
$^c$Measured by FTIR spectroscopy and calculated by the method of [26]. See Appendix A in the Supplemental Material [27].
$^d$y$P$, $V_p$, $K$, and $G$ correspond to $P$- and $S$-wave velocities and bulk and shear moduli, respectively. See Appendix B in the Supplemental Material for details and data tables [27].
$^e$y$P$ = $\frac{4}{3}$.
$^f$K = $\rho (V_p^2 - \frac{2}{3}V_s^2)$.
$^g$G = $\rho V_s^2$. 174110-2
We measured ultrasonic wave travel times at ~0.25 GPa pressure steps from 0 to 5 GPa. P- and S-wave measurements were carried out separately. In several experiments, travel times were also collected on decompression to test for reversibility (Fig. 4 and Appendix C in the Supplemental Material [27]). After each pressure adjustment, data were collected every 10 min with a minimum of 1 h between pressure adjustments to evaluate any time dependence of the travel time measurements. P-wave data for the 985 °C Tf sample includes two compression-decompression cycles (0–4.5 GPa) performed without reloading the sample. P- and S-wave travel times were normalized to the 1-atm travel time (t0) and plotted in Fig. 4. Within uncertainties, the normalized travel times for a given pressure and Tf sample are identical for the compression and decompression paths of a given run, and among replicate experiments.

As shown in Fig. 4, the travel times for both P and S waves increase with pressure up to 2–3 GPa, regardless of Tf. Moreover, this behavior occurs despite the increase in sample density and reduction in sample length, requiring that over this pressure range the compressibility increases with pressure at GHz frequencies, as has been shown previously at MHz frequencies [7,11]. Since we lack direct measurements of sample length and density along our compression/decompression paths, we cannot calculate P- and S-wave velocities, and elastic parameters; however, the normalized travel time curves do constrain the transition from anomalous to normal compression behavior. P-wave travel times reach maximum values near 2.0, 2.2, and 2.4 GPa for 985 °C, 1260 °C, 1500 °C Tf samples, respectively, corresponding to a 12%–13% increase in travel time relative to the 1-atm values (Fig. 4). Normalized S-wave travel times exhibit a similar increase with Tf over this pressure range, although the maximum in travel times occurs ~0.3 GPa higher in pressure for each sample.

We employed two methods to constrain the pressure at which the pressure dependence of the travel time changes from...
FIG. 3. (Top) Interferogram sample $T_f = 985^\circ$C at 0.11 ± 0.05 GPa (985_P1_61) from 0.75 to 1.25 GHz. The dashed curve is the amplitude of the diamond culet signal and the solid curve is the interfered diamond culet–sample signal (see Fig. 2.) Spacing between two maxima corresponds to an integer number of wavelengths in the sample. (Bottom) Calculated travel time as a function of frequency from 0.75 to 1.25 GHz from the above interferogram. The number of wavelengths in the sample is indicated by the $m$ value. Incorrect $m$ values result in frequency-dependent travel times [e.g., $m(1) = 13$ and 15]. Travel time is determined from the $m$ value that provides frequency-independent measurements [e.g., $m(1) = 14$].

FIG. 4. $P$- and $S$-wave travel time curves normalized to the 1-atm travel time shown as circles and squares, respectively. The normalized travel time scale is offset. The intercept is $t/t_0 = 1.00$, with one division = 0.01. Black, gray, and white circles ($P$ wave) and squares ($S$ wave) correspond to $T_f = 985^\circ$C, 1260$^\circ$C, and 1550$^\circ$C data sets, respectively. Decompression data are shown as triangles in contrasting colors for both $P$- and $S$-wave travel time curves (i.e., white and black for $T_f = 985^\circ$C and 1550$^\circ$C, respectively). The dashed line is to guide the eye to the location of the maximum $t/t_0$ measurement. The gray box bounds the pressure region between the maximum travel time location, and the pressure calculated as the intersection of the extrapolated positive (anomalous) and negative (normal) limbs of the travel time curves (see text for discussion, and Appendix C in the Supplemental Material [27] for further details and data tables).

postulated for the flexibility (floppy mode) hypothesis, where the return to normal behavior should occur once the volume reduction reaches the atomic repulsion limit. That is, the $\nu$-SiO$_2$ sample with the higher initial density should reach this locking limit at lower pressure as the atoms are in closest proximity before compression. On the other hand, anomalous compressibility cannot be the consequence of changes in the proportion of LDA and HDA with pressure as proposed by proponents of polymorphism. Firstly, we observe no time dependence or hysteresis in the $t/t_0$ curves during compression and decompression—even following multiple compression-decompression cycles—as would be expected if the time scale...
for phase transitions and/or structural relaxation are less than or equal to the observational time scale. Secondly, given that our study was carried out on glasses having different initial bulk properties, if LDA is to convert to HDA on compression at room temperature one expects the two-way travel time curves to converge at high pressure—which is not observed at least up to 4.7 GPa. In summary, the reproducibility and reversibility of $t/t_o$ curves coupled with differences related to $T_f$ imply that the proportion of LDA and HDA in a given sample remains constant during room temperature compression and decompression. While the anomalous elastic behavior of $\nu$-SiO$_2$ presented here requires the presence of both LDA and HDA, it cannot arise from an active transition between the two amorphous states.

The exact proportions of LDA and HDA in our samples is not known, but the 1-atm $T$-$\rho$ relations reflecting a negative Claperyon slope for the LDA-HDA spinodal field [21] provides guidance. The sample with a $T_f$ of 985 $^\circ$C reflects annealing close to the density minimum and thus possesses the greatest amount of LDA, while the sample with a $T_f$ of 1500 $^\circ$C and annealed closer to the high-temperature density maximum has the least amount of LDA. We estimate that the sample with a $T_f$ of 1260$^\circ$C lying about halfway between the extrema in density has subequal proportions of HDA and LDA. Although the precise volumetric proportions of LDA and HDA in our samples are not known, importantly there is a systematic shift in the location of the maximum in the $t/t_o$ curves to higher pressure with increasing $T_f$ (Figs. 4 and 5). This implies that the return to normal behavior shifts to higher pressure as the initial proportion of HDA increases. This interdependence of $T_f$, abundance of LDA and HDA, and pressure at which $\nu$-SiO$_2$ reverts to normal (decreasing) two-way travel times with increasing pressure provides persuasive evidence that LDA is more compressible than HDA. That is, LDA being initially less dense than HDA must reach a locking limit at lower pressure. Supporting this interpretation is the positive dependence of the 1-atm bulk modulus ($K_S$) on $T_f$ (density and proportion of HDA) shown in Fig. 6. The near constancy of $G$ (∼31 GPa) may largely be fortuitous reflecting an offsetting effect of differences in the abundance and shear moduli for LDA and HDA.

We propose that the observed $T_f$-$t/t_o$ relationships reflect network flexibility within a two-phase mixture that have fixed proportions up to at least 5 GPa, with normal behavior occurring upon the collapse of the network to a critical density where the repulsive atomic forces dominate energetically over tetrahedral rotation. The fact that all the samples exhibit the same magnitude of increase in the $t/t_o$ curves requires that the mechanism underlying the anomalous behavior is active in both LDA and HDA phases—the differences we show here with $T_f$ simply reflect the relative abundance of the two phases and their respective $K_S$. If the proposed mechanism was active only in one phase, the magnitude of the increase in the $t/t_o$ curve should be dependent on the proportion of phases present in the sample, which is not observed in our data. This does not imply that the LDA and HDA phases have the same topology, but that the manner of network collapse is similar in both phases. LDA (low $K_S$) and HDA (high $K_S$) phases both show anomalous behavior on compression, in direct contrast to the polymorphism model, which predicts that both the LDA and HDA behave normally on compression and that it is the transition from the low-compressibility LDA to the higher-compressibility HDA that leads to the anomalous behavior.

IV. CONCLUSION

In summary, the results presented here provide evidence for polymorphism in liquid silica that upon quenching results in glasses with frozen-in LDA and HDA phase assemblages. The relative volumetric proportions of these phases depend on the thermal history of the glass with the fraction of the HDA increasing with $T_f$. Regardless of $T_f$, $\nu$-SiO$_2$ shows anomalous compressibility between 0 and ∼3 GPa, consistent with previous studies; however, this work shows in detail that the elastic properties of $\nu$-SiO$_2$ vary systematically with $T_f$, i.e., the proportions of LDA and HDA. We attribute the
anomalous compressibility to floppy mode tetrahedral rotation, i.e., the flexibility hypothesis, but in a two-phase mixture, consistent with the proposed structural differences between HDA and LDA phases. The difference in bulk compressibility among glasses with different $T_f$ is consequently explained by the relative fraction of these phases. The observed behavior requires that LDA is more compressible than HDA in order for LDA to reach the repulsive limit and return to normal behavior at lower pressure. Thus, both floppy modes and the coexistence of varying fractions of HDA and LDA phases are critical factors controlling the elastic properties of $\nu$-SiO$_2$, and may be the underlying cause of anomalous compressibility found in other network glasses and melts [11,31].

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