2 6

Proc. Natl. Acad. Sci. USA
Vol. 78, No. 1, pp. 20–21, January 1981
Geophysics

On the importance of changes in the gravity field on seismic recording at ultralong periods

(seismology/normal modes/theoretical geophysics)

EMILE A. OKAL

Department of Geology and Geophysics, Yale University, Box 6666, New Haven, Connecticut 06511

Communicated by John Rodgers, September 25, 1980

ABSTRACT **The effect of changes in gravity induced by the Earth's deformation on seismic recording at ultralong periods is studied quantitatively for the low-order spheroidal modes of the Earth. Because this effect can either enhance or reduce the recorded amplitude of a mode, depending on its geometry, it may become nontrivial at the longest free-oscillation periods, and contribute to relative deviations of up to 17%.**

At ultralow frequencies, most instruments recording the vertical component of the Earth's displacement actually operate as accelerometers. Some of them are specifically designed to record accelerations [e.g., the I.D.A. instruments (1)], but any vertical pendulum starts behaving as an accelerometer once the excitation frequency becomes much lower than its eigenfrequency, reflecting the fact that the instrument merely follows the slow changes in the apparent gravity field of the accelerated frame linked to the oscillating surface of the Earth. The response of the sensor in the instrument is then proportional to $\omega^2 \cdot u(\omega)$, where ω is the angular frequency of oscillation and $u(\omega)$ is the spectral amplitude of the Earth's vertical displacement at that frequency. However, in deriving this response, one usually neglects any variations of the gravity field of the planet brought about by the seismic disturbance. These variations contribute to the excitation of the pendulum by changing its equilibrium position and similarly affect any other recording system by contributing an additional field of acceleration.

In the formalism of normal mode theory, the change of gravity can be written as $\Gamma \cdot Y_\ell^m$, where Y_ℓ^m is the spherical harmonic of the spheroidal mode. It is directly proportional to the excitation of the spheroidal mode and, therefore, to $u(\omega) = y_1 \cdot Y_\ell^m$, in which y_1 is the vertical eigenfunction of the mode at the surface and the notation of Saito (2) is used. This effect is clearly trivial in the traditional seismic frequency band, but at ultralow frequency, as ω goes to 0, Γ may become important relative to $y_1 \cdot \omega^2$. However, no quantitative assessment of this contribution is available in the literature. The purpose of this study is to discuss quantitatively the importance of this gravity term for the longest-period spheroidal modes.

The variation of the gravity potential is an integral part of any computation of the spheroidal oscillation of a self-gravitating Earth. Pekeris and Jarosch (3) have shown that the resulting change in the gravity field $\Gamma \cdot Y_\ell^m$ at the surface ($r = a$) is given by:

$$\Gamma = -\frac{dy_5}{dr} = -4\pi G\rho_a y_1 - \frac{\ell + 1}{a} y_5, \qquad [1]$$

where G is the gravitational constant, ρ_a is the undeformed surface density, Γ is positive downwards, and the notation of Saito (2) is used. The response of a pendulum (or of an accelerometer such as the I.D.A. instrument) is then governed by the quantity $y_1\omega^2 - \Gamma$ rather than by the simple seismic acceleration $y_1\omega^2$. To account for the influence of gravity, instrument responses routinely used in seismogram synthesis should be multiplied by

$$\alpha = 1 - \frac{\Gamma}{y_1\omega^2}. \qquad [2]$$

Fig. 1 shows a plot of $(\alpha - 1)$ as a function of frequency for all spheroidal modes with periods larger than 300 sec, obtained from a set of eigenfunctions computed for Earth model 1066A (5). Core and Stoneley modes [the so-called K modes in Okal's classification (4)] have been omitted because they contribute no significant motion to the Earth's surface. It is clear from the figure that the effect under study is extremely small except for periods greater than about 15 min. Therefore, it could affect only investigations of extremely long components of seismic sources, such as the reported precursor to the Chilean earthquake of 1960 (6).

The first term in Eq. 1 represents a free-air anomaly and expresses the fact that the point of observation changes altitude during the seismic displacement. The second term is a Bouguer term and is related to the changes in mass distribution accompanying the oscillation of the Earth. For modes involving primarily radial displacement [Okal's "V" modes (4)], the Bouguer term is negligible (identically zero for the radial modes $\ell = 0$), and

$$\alpha - 1 = 4\pi G\rho_a/\omega^2. \qquad [3]$$

The change in gravity would equal the seismic acceleration at a theoretical period $T = 0.5\sqrt{\pi/G\rho_a} = 4653$ sec, way beyond the seismic spectrum of V modes, but the effect of gravity remains on the order of a few percent for the fundamental "breathing" mode ${}_0S_0$ (${}_0V_0$), whose period is 1230 sec. The free-air anomaly curve is shown as a dashed line on Fig. 1.

For other modes, such as the "football" mode ${}_0S_2$ (${}_0R_2$), the Bouguer term is prominent and may result in a decrease in the response of the instrument on the order of 9% for ${}_0S_2$. Table 1 is a summary of the six modes for which $|\alpha - 1|$ is greater than 5%. Among modes currently observed after large earthquakes, only ${}_0S_2$, ${}_1S_2$, and ${}_0S_0$ are significantly affected. ${}_2S_1$ is strongly affected but hardly excited by any regular seismic source. As for ${}_2S_5$, the high value of α is due to its having a mode of vertical displacement close to the surface ($y_3 >> 1$).

An important result from Table 1 and Fig. 1 is that the effect of gravity can either enhance or reduce the recorded amplitude of the various modes; although we doubt that the present results affect significantly any past studies of normal mode observations, relative deviations in the apparent excitation of spheroidal modes of up to 17% may result from neglecting the influence of

The publication costs of this article were defrayed in part by page charge payment. This article must therefore be hereby marked "*advertisement*" in accordance with 18 U. S. C. §1734 solely to indicate this fact.

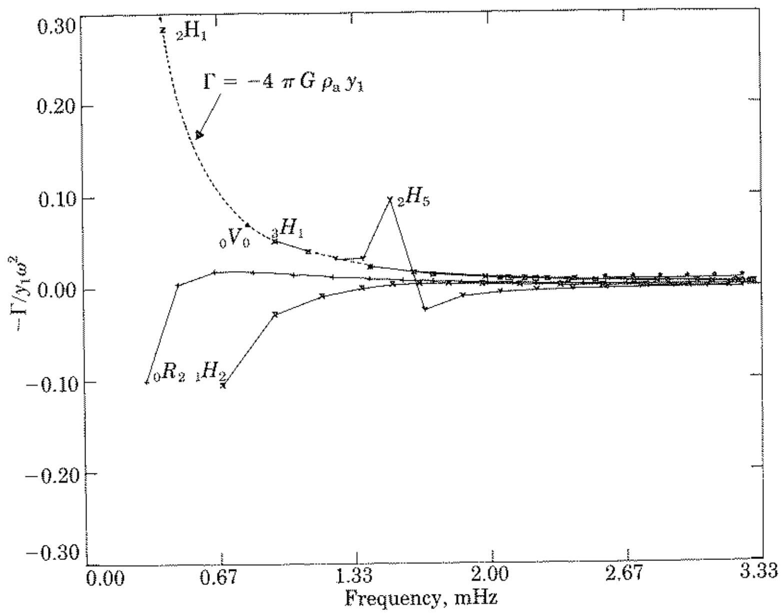

FIG. 1. Plot of $\alpha - 1 = -\Gamma/y_1 \cdot \omega^2$ as a function of frequency for all spheroidal modes with a period greater than 300 sec. Core and Stoneley modes are excluded. ---, The theoretical curve of free-air anomaly, on which all radial modes sit; ——, pseudo-overtones as defined by Okal (4). Different symbols are used to distinguish between pseudo-overtone branches.

mode-induced gravity changes in the local acceleration field. A particularly interesting feature concerns $_1S_3$ and $_3S_1$ ($_1H_3$ and $_3H_1$), two modes with very similar periods and corrections $\alpha - 1$ of opposite signs, contributing deviations differing by about 10% in their relative excitation; the effect of Γ (which in zeroth order can be approximated by its values for the unperturbed multiplets) then becomes comparable to the various components of hybridization between the two modes induced by ellipticity (7) and, therefore, may not be considered as totally trivial.

Table 1. Spheroidal modes* for which $|\alpha - 1| \geq 0.05$

Mode nomenclature			
1066A (5)	Okal (4)	Period(s)	α
$_0S_2$	$_0R_2$	3232	0.90
$_2S_1$	$_2H_1$	2476	1.28
$_1S_2$	$_1H_2$	1470	0.90
$_0S_0$	$_0V_0$	1230	1.07
$_3S_1$	$_3H_1$	1061	1.05
$_2S_5$	$_2H_5$	661	1.10

* Core and Stoneley modes excluded.

I thank Seth Stein and Robert Geller who offered constructive reviews of an earlier draft of this paper. I am grateful to Freeman Gilbert for a complete set of 1066A eigenfunctions. This research was supported by National Science Foundation Grant EAR79-03907.

1. Agnew, D. C., Berger, J., Farrell, E. & Gilbert, F. (1976) *Trans. Am. Union* **57**, 180–188.
2. Saito, M. (1967) *J. Geophys. Res.* **72**, 3689–3699.
3. Pekeris, C. & Jarosch, H. (1958) *Int. Ser. Monogr. Earth Sci.* **1**, 171–192.
4. Okal, E. A. (1978) *J. Phys. Earth* **26**, 75–103.
5. Gilbert, J. F. & Dziewonski, A. M. (1975) *Philos. Trans. R. Soc. London Ser. A* **278**, 187–269.
6. Kanamori, H. & Cipar, J. J. (1974) *Phys. Earth Planet. Inter.* **9**, 128–136.
7. Woodhouse, J. H. (1980) *Geophys. J. R. Astron. Soc.* **61**, 261–283.