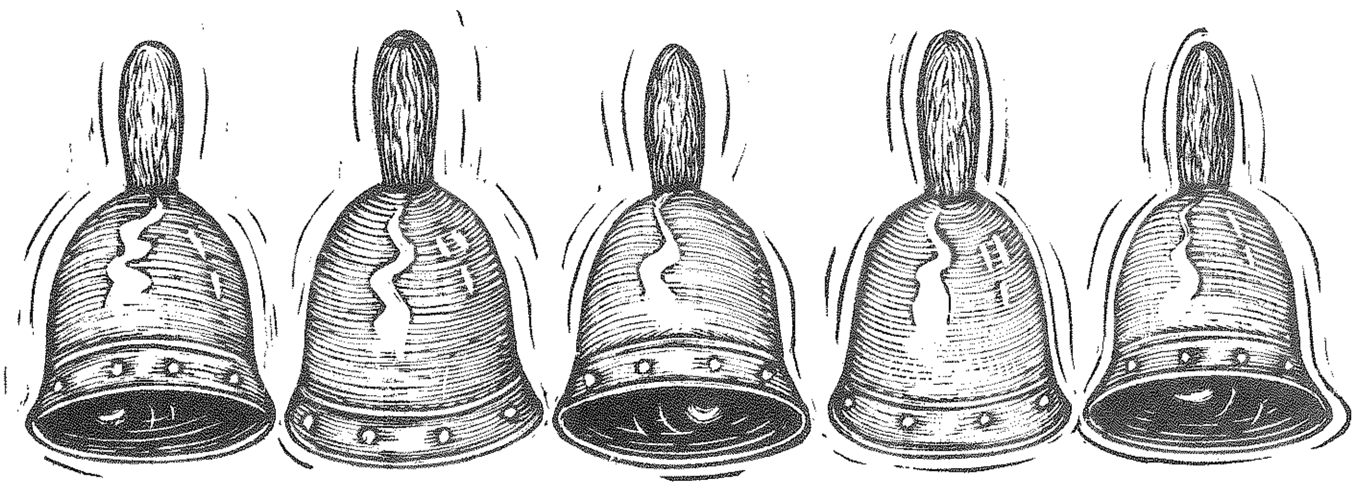
LISTENING TO THE EARTH'S MUSIC

by Emile A. Okal
Yale University

As a finite elastic body, the Earth can ring like a bell when excited by a large earthquake; modeling the modes of oscillation provides insight into the deep structures of the planet and the sources of earthquakes



The ability of musical instruments to generate a discrete sequence of pitches resides in their finite geometrical size, which governs the frequencies at which they can oscillate freely. Seismic waves, traveling through the Earth after an earthquake or a large nuclear explosion, are similar to sound waves. Therefore, it is no surprise that the Earth, as a finite body, should have a discrete number of normal modes of oscillation. But in studying the Earth, because of its very large size, the physical law relating the dimensions of a vibrating body to its frequencies of oscillation must be extrapolated several orders of magnitude, characteristically to periods on the order of ten minutes to an hour.

Precise measurements of the frequencies of the Earth's normal modes, and of their excitation by large earthquakes, have provided geophysicists with a wealth of information about the internal elastic and anelastic structure of the Earth, as well as with a clearer view of the mechanism of energy release in major earthquakes. It has been only during the last two decades that such measurements have been possible. Thanks largely to the availability of high-speed computing and digital recording technology, it has been possible to establish a network of ultra-low-frequency seismic stations to record the Earth's free oscillations. Satellite telecommunications make it feasible to record, process, and analyze source characteristics in real time. These advances should make it possible to provide early warning against the devastating sea waves known as *tsunamis,* which are generated by major earthquakes in oceanic areas.

In simple terms, the problem is that of finding a regime of vibration of the planet's interior which satisfies both the constitutive properties of the rock...and a "boundary condition" stating that stresses at the Earth's surface must vanish in order to match the atmospheric pressure.

Emile Okal is an associate professor, Department of Geology and Geophysics, Yale University, Box 6666, New Haven, Connecticut 06511.

The Earth's free oscillations

Long before seismology was born as a science late in the nineteenth century, the theory of elasticity was established, notably by the French scientists Navier and Poisson in the 1820s. A number of investigators attacked the then purely theoretical problem of finding the pitch (or "eigenfrequencies") of the Earth's free oscillations. Lacking computing power, as well as knowledge of the physical nature of the Earth's interior, Lamb[1] modeled the planet simply as a homogeneous sphere made of steel. He computed a period of 78 minutes for the gravest mode of the Earth. Taking into account the Earth's self-gravitation, Love[2] later revised that figure to an even 60 minutes. We now know that the Earth has a strongly differentiated structure: a silicate crust and mantle extending roughly to a depth of 2900 kilometers, overlying a molten core made primarily of iron, with a solid inner core, most probably consisting of an iron-nickel alloy.

It was not until the 1950s that progress in seismic instrumentation enabled Benioff[3] to observe free oscillations, after the 1952 Kamchatka earthquake (Figure 1), at a period amazingly close to Love's estimate: 58 minutes. A figure of 3232 seconds (53.9 minutes) is now generally accepted. By the time of Benioff's observation, studies of seismic body waves propagating through the Earth's interior had led to a good knowledge of its structure. Benioff's work stirred interest among theoretical geophysicists, who eagerly started devising methods of computing the Earth's normal modes for representative models of the Earth's structure. Pekeris and Jarosch[4] showed in 1958 that, in the most general case, the free oscillations are obtained as the solutions of a first-order differential system in a six-dimensional vector space, satisfying a set of boundary conditions in each spherical layer in which the Earth's structure can be considered homogeneous. With the aid of data processing technology, geophysicists have developed efficient codes, based principally on Rayleigh-Ritz variational procedures,[5] for finding the Earth's normal modes, given a structural model. Similarly, they have developed codes for solving the inverse problem of refining the structure of the Earth, given observations of its free oscillations.

In simple terms, the problem is that of finding a regime of vibration of the planet's interior which satisfies both the constitutive properties of the rock at all depths and a "boundary condition" stating that stresses at the Earth's surface must vanish in order to match the atmospheric pressure. This boundary condition, similar to that governing the pitch of an organ pipe, dictates the frequencies of the Earth's modes. The actual computation uses a layered model consisting of as many as 120 concentric shells. It assumes an estimate of the mode frequency, and it computes a mode of vibration which, however, is imperfect in that it satisfies all but one physical condition—that kinetic and potential energies be equal. It then uses the value of the misfit between these two forms of energy to refine the estimate of the frequency, and the process is iterated. Convergence to six-digit accuracy usually is achieved in about five iterations, which are performed automatically.

Among the most important scientific results obtained from the systematic study of the Earth's normal modes, one can list decisive evidence of the solidity of the planet's inner core.

Geometry. Geophysicists distinguish two fundamentally different families of oscillations, called spheroidal (or poloidal) and torsional (or toroidal) modes. The type of oscillation depends on the symmetry of the Earth's deformation in the vibration. Torsional modes correspond to shear deformation, which cannot penetrate the deep, liquid outer core of the planet. The simplest (and gravest) torsional mode, with a period of 2631 seconds, is represented in Figure 2A. A single oscillation can be visualized as a twisting of the northern hemisphere eastward and the southern hemisphere westward; during the next oscillation, the motion is reversed. Other torsional modes can be visualized by increasing the number of latitudinal bands in this pattern, as well as by including a longitudinal dependence of the pattern (Figures 2B and 2C). Overtones, similar to those giving richness to the sounds produced by a musical instrument, can exist, depending on whether the pattern observed at the Earth's surface is valid at all depths, or whether and how often it changes with depth.

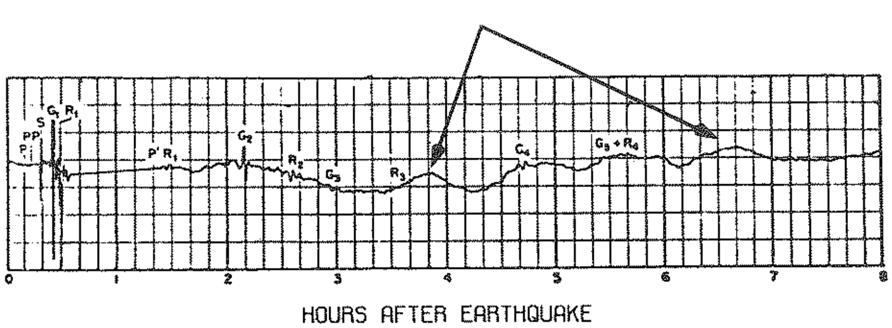


Figure 1. The first observation of the free oscillations of the Earth was made by Benioff[3] on this strainmeter record of the 1952 Kamchatka earthquake. Arrows show the oscillation of the planet in the "football" mode, with a period of a little less than an hour.

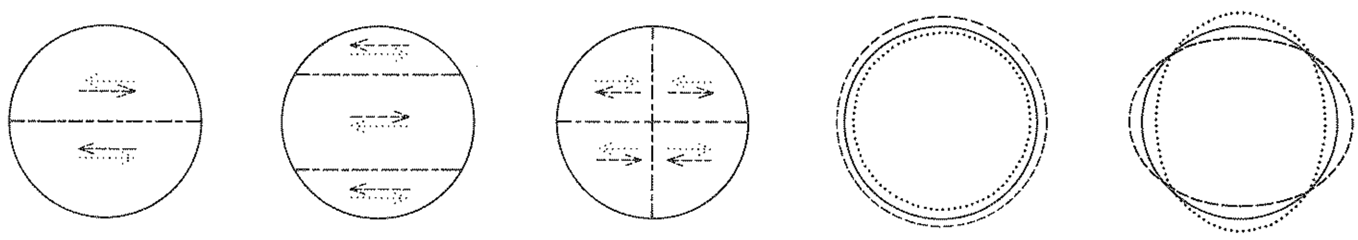


Figure 2. Geometry of vibrations for a few simple free oscillations of the Earth. In each case the direction of vibration, or the shape of the deformed Earth during the first oscillation, is shown as a dashed line. Effects of the next oscillation are shown as dotted lines. The undeformed Earth is shown in color.

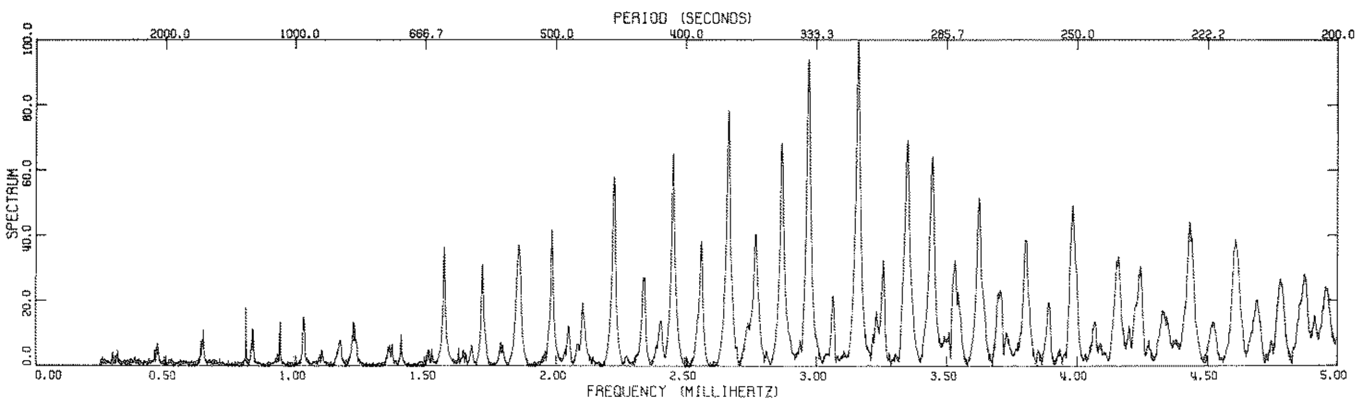


Figure 3. Spectrum of the spheroidal normal modes of the Earth obtained from the 1977 Indonesian earthquake at Rarotonga (Cook Islands, South Pacific). Each peak represents a mode of vibration of the Earth. The combined "ringing" of all the individual peaks produces the seismic displacement "heard" by the recording instrument.

Spheroidal modes, on the other hand, are more complex. They involve deformation in shape, rather than deformation by twisting. The simplest spheroidal mode is the "breathing" mode, represented in Figure 2D, in which the Earth "inflates" during one oscillation and "deflates" during the next, with a period of about 20.5 minutes (1229 seconds). The gravest spheroidal mode is the "football" mode (Figure 2E) in which the polar regions rise at

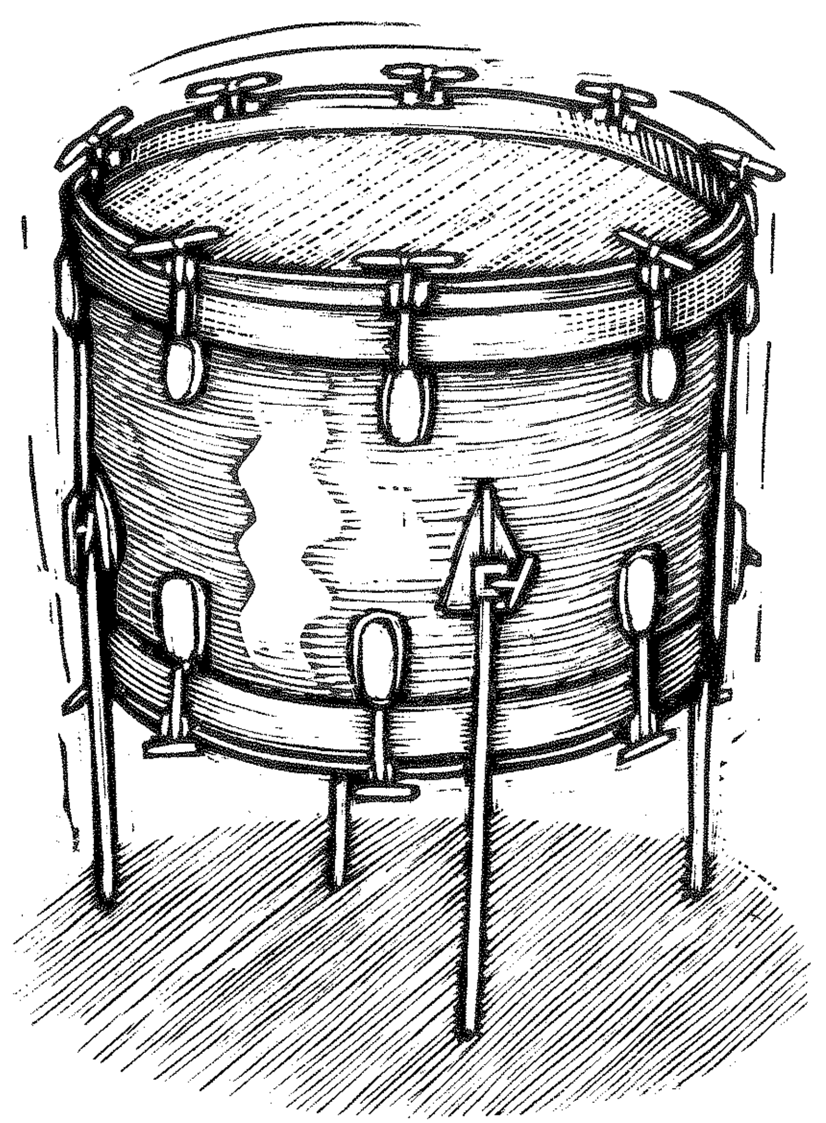
The study of how records of normal modes differ slightly...yields important data on how deep in the mantle structural differences (for example, continents versus oceans) extend.

the expense of the equator during one oscillation, and then, during the next, subside while an equatorial bulge is developed. This is the Earth's gravest mode, with a period of 3232 seconds (almost 54 minutes). As with torsional modes, other spheroidal modes involve more complex surface patterns and radial variations of the deformation.

A three-index nomenclature ${}_nS_l^m$ (spheroidal) or ${}_nT_l^m$ (toroidal) keeps track of the complexity of the vibration from pole to pole (l), azimuthally (m), and radially inside the Earth (n). If there is no azimuthal dependence, the symbol m usually is dropped. It should be emphasized that the "poles" referred to can be arbitrary points on the globe. (Similarly, the "equator" and values of latitude and longitude are arbitrary.) In practice, the pole will be located at the epicenter of the parental earthquake. Weak perturbations are contributed to this general framework by the Earth's slight ellipticity, its diurnal rotation, and its slight superficial lateral heterogeneity (for example, continental structure compared with oceanic structure).

Early developments

Normal mode theory made considerable progress in the 1960s, following the earthquakes of 1960 in Chile and 1964 in Alaska. These truly gigantic events provided a wealth of spectral data on the Earth's free oscillations, of the type shown in Figure 3, although of a lower signal-to-noise quality. During the same period, theoreticians attacked the problem of the excitation of an individual mode by a given earthquake of known geometry and size (we will avoid the concept of magnitude, which, although widely used, is poorly suited



for measuring the gigantic earthquakes that can trigger the Earth's normal modes). This problem is equivalent to the following one in music: Given a quantitative kinematic description of the motion of a musician's fingers, find not only the pitch but the loudness (in decibels) at which the instrument (say a guitar) will play.

Models for the kinematic description of an earthquake source had become available following advances in body-wave seismology. The contributions of Jobert,[6] Saito,[7] Gilbert,[8] and Kanamori[9] led to the use of synthetic seismograms to model the motion of the Earth at any distance from a major earthquake by superimposing a very large number of free oscillations (see Figure 4). Conversely, developments in inversion theory led to the exploration of the behavior of seismic sources at ultralong periods. This work led seismologists to propose that some (but certainly not all) earthquakes could exhibit very slow components in their mechanisms, some of which may have a premonitory character.[10,11] If confirmed and identified, such precursory "slowquakes" may one day play a role in the difficult art of earthquake prediction.

In the end, this better picture of the material properties of the deep structures of the Earth...will help explain the processes of differentiation undergone by the Earth and other planets about four billion years ago.

Among the most important scientific results obtained from the systematic study of the Earth's normal modes, one can list decisive evidence of the solidity of the planet's inner core. Under certain circumstances when two normal modes have nearly similar periods, they can be coupled to each other, leading to substantial changes in the Earth's vibration. In 1973 Dziewonski and Gilbert[12] observed such a pattern for the mode ${}_{10}S_2$ in the spectrum of the Alaskan earthquake of 1964. This situation could be explained only by hybridization with a shear mode of the inner core, whose mere existence requires it to be solid. Although the existence of the inner core had been proved in 1936 by Inge Lehmann, and it had long been thought to be solid, all attempts at identifying shear waves (which are characteristic of solids) in the inner core had until 1973 either failed or, at best, been shaky.

Another area where normal mode theory has made considerable inroads is in investigations of the structure of the planet's interior. The study of how records of normal modes differ slightly, depending on where on the Earth's surface they are taken, yields important data on how deep in the mantle structural differences (for example, continents versus

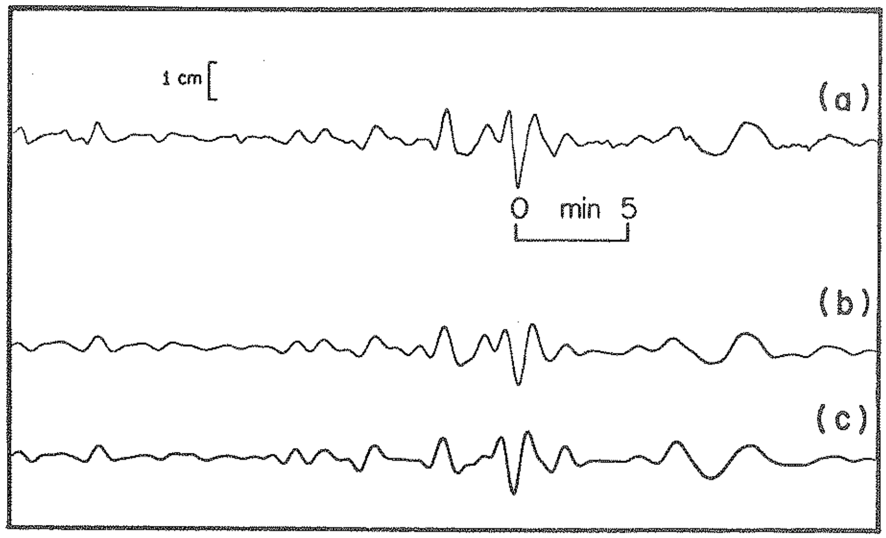
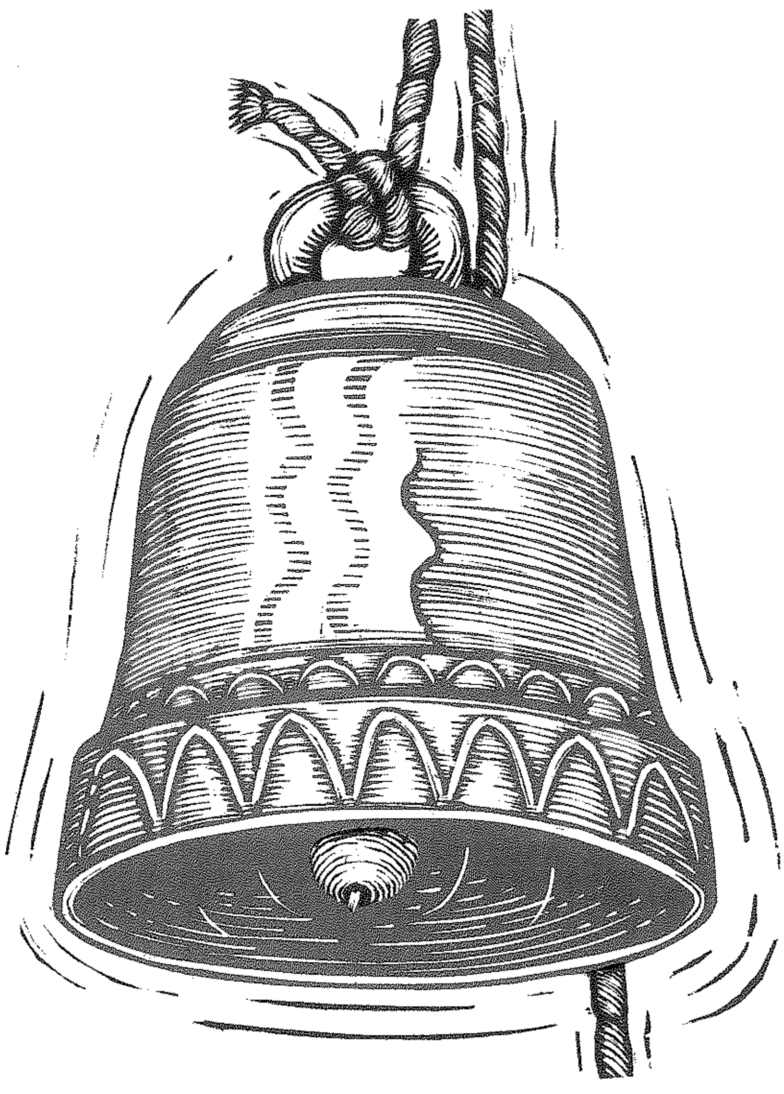


Figure 4. Example of modeling of seismograms using normal mode theory: (a) original trace of a seismic record of a deep Japanese earthquake at Pasadena, California; (b) result of low-pass filtering to eliminate local noise; (c) synthetic seismogram made by superimposing a large number of free oscillations (in this case, 2979). This superimposition can be compared with the symphony effect obtained by superimposing numerous overtones from many musical instruments.

oceans) extend. That depth provides constraints on the possible patterns of convection that are thought to drive the cycle of plate tectonics, which is responsible for the opening of ocean basins and the drifting of continents. Present results, although controversial, indicate probably 250 kilometers as the maximum depth at which substantial structural heterogeneities are present.[13-15]

Finally, a further area of study of the Earth's free oscillations is their attenuation, or how fast they die out after being excited. Any musical instrument, such as a bell, a piano string, or a drum, will vibrate for only a limited time before its sound falls below the level of ambient noise and cannot be detected. This is because anelastic attenuation converts the acoustic energy into heat waste. The Earth is no exception, and its normal modes are damped by attenuation. This is characterized by a "quality" factor Q, which in simple terms indicates how many periods of oscillation will evolve before only 0.2 percent of the mode's initial energy remains available for elastic oscillation. The least attenuated mode by far is the "breathing" mode ${}_0S_0$ (because attenuation affects mostly shear motion, and ${}_0S_0$ has very little shear), with a Q value of about 4800. Two weeks after the large Indonesian earthquake of 1977, the "breathing" of the planet was still being recorded, at an amplitude of approximately two microns (2×10^{-4} centimeters).



The Q value of a mode can be measured from the width of the mode's spectral line in a frequency spectrum, such as the one shown in Figure 3. Little can be learned about the deeper layers through standard techniques,[16] so a large effort is now under way to obtain reliable measurements of mode Q values, especially for overtone modes, which sample the planet at great depths and are thus sensitive to the anelastic properties of deep layers. In the end, this better picture of the material properties of the deep structures of the Earth, in particular the lower mantle and the inner core, will help explain the processes of differentiation undergone by the Earth and other planets about four billion years ago.

The IDA network

By 1970 it had become clear that existing seismic instruments, designed to record body waves and surface waves with periods of no more than 100 seconds, were not appropriate for the study of the Earth's modes. Normal-mode seismologists at that time were in a position similar to that of a person listening to a high-fidelity re-

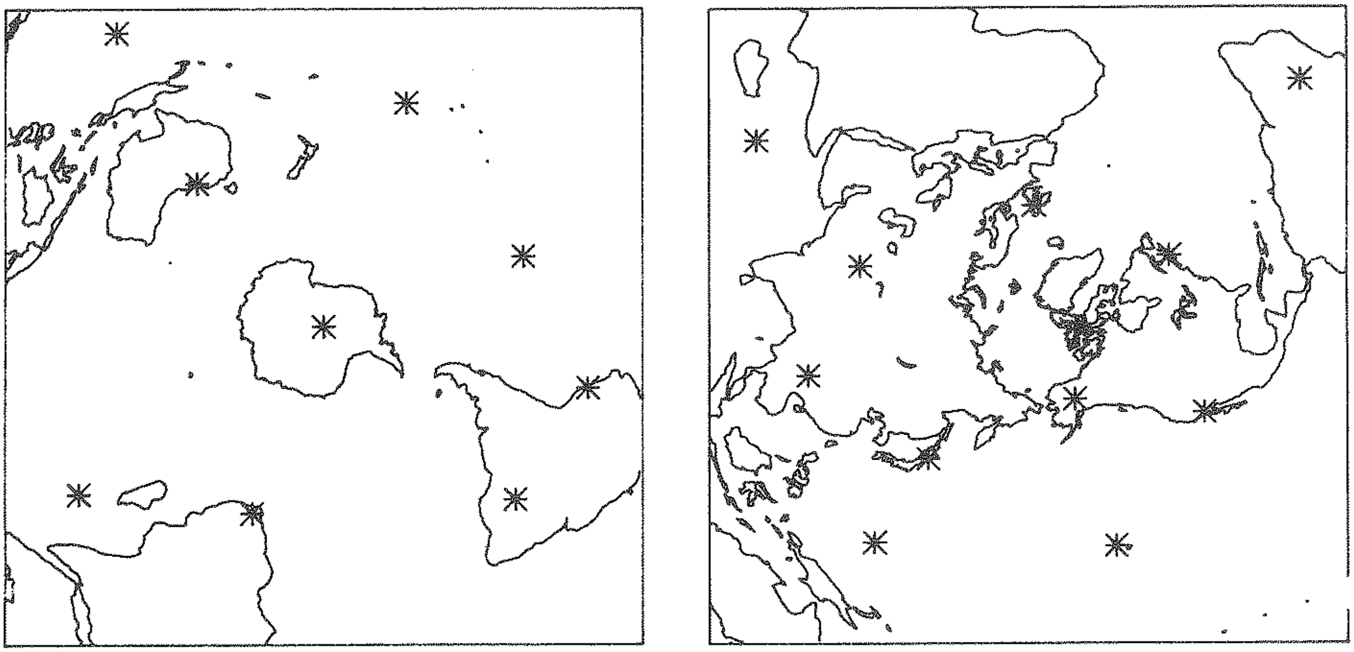


Figure 5. Computer-generated maps showing locations of the IDA stations (asterisks).

cording over a telephone line designed to carry only a small range of human voice frequencies. Clearly, new instruments were needed to take full advantage of the new theory. Thus, starting in 1975, a completely new seismic network known as IDA (for International Deployment of Accelerometers) was established around the world, as shown in Figure 5.

Because of the very large wavelengths involved at ultralow frequencies, only twenty IDA stations are sufficient to provide worldwide coverage.[17] The sensors are basically Lacoste-Romberg gravimeters, which measure changes in the Earth's apparent gravity and therefore in its vertical acceleration, which is the second time derivative of the seismic displacement. Sampling takes place every ten seconds, and recording is on magnetic tape in cassettes with a capacity of about 100 000 twelve-bit data points. The cassettes, which last for a week to ten days, are airmailed regularly to a central processing facility (presently at the University of California at San Diego). There the data is entered into computer storage and reformatted on standard tapes, which are made available to seismologists for research.

Computer software, developed mostly by individual scientists engaged in research on IDA data, allows automatic scanning of tapes, once an earthquake's time of origin is given (the first record of each file on the tape is a header that gives the date and station information), followed by the extraction of an adequate time window—typically one week—and signal analysis. In its simplest form, an analysis consists of fast Fourier transforming (of up to 2^{15} points), resulting in spectral-amplitude plots similar to the plot in Figure 6. The spectra can also be stored for further

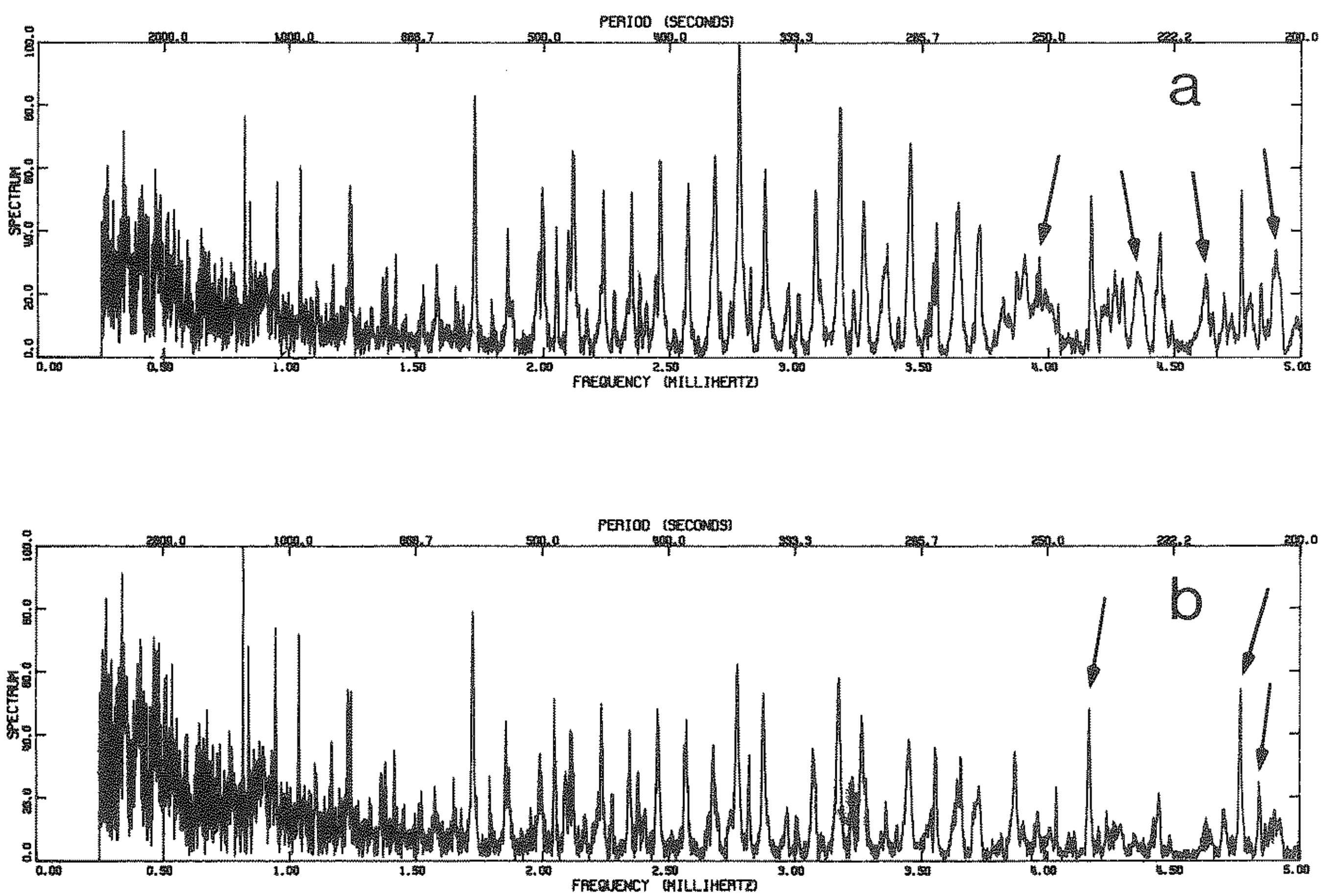
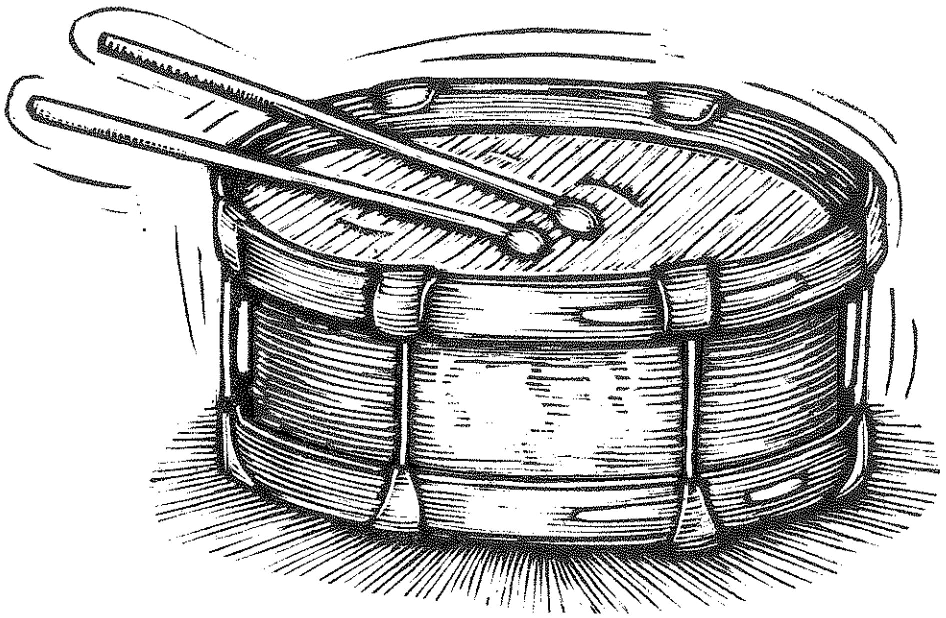


Figure 6. Earth spectra obtained from IDA records of the 1977 Tonga Islands earthquake at Sutherland, South Africa. Comparison with Figure 3 shows a richer spectrum due to greater depth of the earthquake. Frame b, obtained from a window starting seven hours after the event, highlights high-Q overtones, especially at low frequency (arrows), where fundamental mode energy has been attenuated by anelasticity.



study, as in measuring Q values. In determining Q values, the mode is first identified using a computerized catalog of frequencies; the modal line is then isolated in the frequency domain, and the Q value is retrieved by performing a least-squares fit to theoretical line shapes. Plotting the fit interactively at a terminal enables the scientist to refine the measurement.

The main difference between Figure 6 and Figure 3 is the depth of the parental earthquake. Figure 3 presents the spectrum of a shallow earthquake (depth of focus, 40 kilometers) which excited principally fundamental modes and few if any overtones. Figure 6 illustrates a deeper event (150 kilometers) which excited a wealth of overtones. The upper frame (a) of Figure 6 represents a spectrum taken immediately after the earthquake. At right in the upper frame, in the 4- to 5-millihertz frequency range, there are many fundamental modes with comparable amplitudes and relatively broad spectral lines (indicated by arrows). On the other hand, if the analysis is started seven hours later, these modes will have lost their energy to anelastic attenuation as shown in the lower frame (b), and their amplitude will have fallen to the ambient noise

A systematic search for "slow-quakes" could be of great help in the continuous monitoring and potential prediction of seismicity.

level. This spectrum highlights a few distinctive overtones (arrows, lower frame), whose higher Q values, clearly evidenced by their much narrower spectral lines, have made them much more resistant to the toll of anelastic attenuation.

Other acoustical phenomena can be observed in the frequency range of the Earth's normal modes, in particular the phenomenon of beating between two or more modes of very similar periods. Beating can be produced, for example, by the splitting that the Earth's rotation introduces between modes of otherwise identical periods. An example is given for $_0S_2$ in Figure 7. Recognition of this phenomenon and its exact quantification by Stein and Geller[18] have greatly contributed to improving our understanding of the attenuation of some of the Earth's gravest modes.

Slow earthquakes and tsunami warnings

The development and refinement of inversion theory have led to the development of efficient computer programs for rapid determination of earthquake source parameters from IDA data.[19] Basically, these programs analyze long-period surface waves (which can be considered superpositions of Earth modes). The difference in seismic amplitudes recorded at various stations is measured directly from digital records and translated into a "radiation pattern" at the epicenter, which represents the variation in azimuth of the amplitude of seismic waves generated by the earthquake. (In simple cases, the seismic radiation pattern can have the shape of a clover leaf; it has no simple equivalent in acoustics but would be comparable to the lobes associated with a radio antenna.) The shape of the radiation pattern is interpreted in terms of the geometry of faulting (azimuth and dip of the fault plane and direction of slip along the fault), while its amplitude gives a measurement of the size of the earthquake source.

The programs have been useful in calibrating major earthquakes in a domain of periods and a range of magnitudes where conventional magnitude scales, such as Richter's, become saturated because of interference effects at the source. In particular, they have shed new light on the existence of slow earthquakes. In addition, through measurement of a quantity called the *seismic moment*, representative of seismic energy released at ultralow frequencies, the programs have allowed the size of gigantic earthquakes to be evaluated correctly.

Present efforts in earthquake prediction are directed at understanding the elastic phenomena that precede rock fracture and catastrophic strain release. The identification of slow earthquakes, and careful studies of the long-period mechanism of major earthquakes using the techniques discussed above, are important steps in this direction. A systematic search for "slowquakes" could be of great help in the continuous monitoring and potential prediction of seismicity.

Additionally, it has been shown that the above-mentioned seismic moment is one of the quantities governing the excitation of tsunamis, the slow-propagating sea waves that are often but improperly called tidal waves (they have nothing to do with tides). Tsunamis can have devastating effects when they reach coastal areas. Chile, Japan, Hawaii, and other areas, especially around the Pacific Ocean, have a history of catastrophic tsunamis. Tsunamis are created by large earthquakes in or near oceanic areas. They can be visualized as large-scale ripples resulting from the shaking of the ocean's bottom. Because tsunamis propagate relatively slowly (at about 240 meters per second), they require several hours to travel any substantial distance. Thus there is a possibility of providing tsunami warnings and evacuating low-lying areas after the parental earthquake has been detected. Unfortunately, the tsunamigenic character of an earthquake is poorly described by the conventional magnitudes readily reported by seismic observatories. On the other hand, Abe[20] has proved that the seismic moment of the parental earthquake is a reliable measure of its tsunami potential.

Tsunamis are also subject to gigantic directivity effects which, by focusing the energy of a wave in one direction from its source, can add an order of magnitude to its amplitude and consequently increase its potential for devastation. The direction of maximum tsunami radiation is perpendicular to the direction of rupture propagation during the parental earthquake.[21] Kanamori and Given[19]

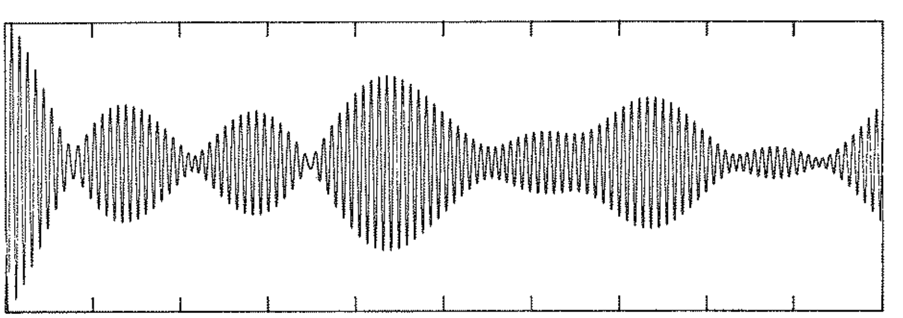
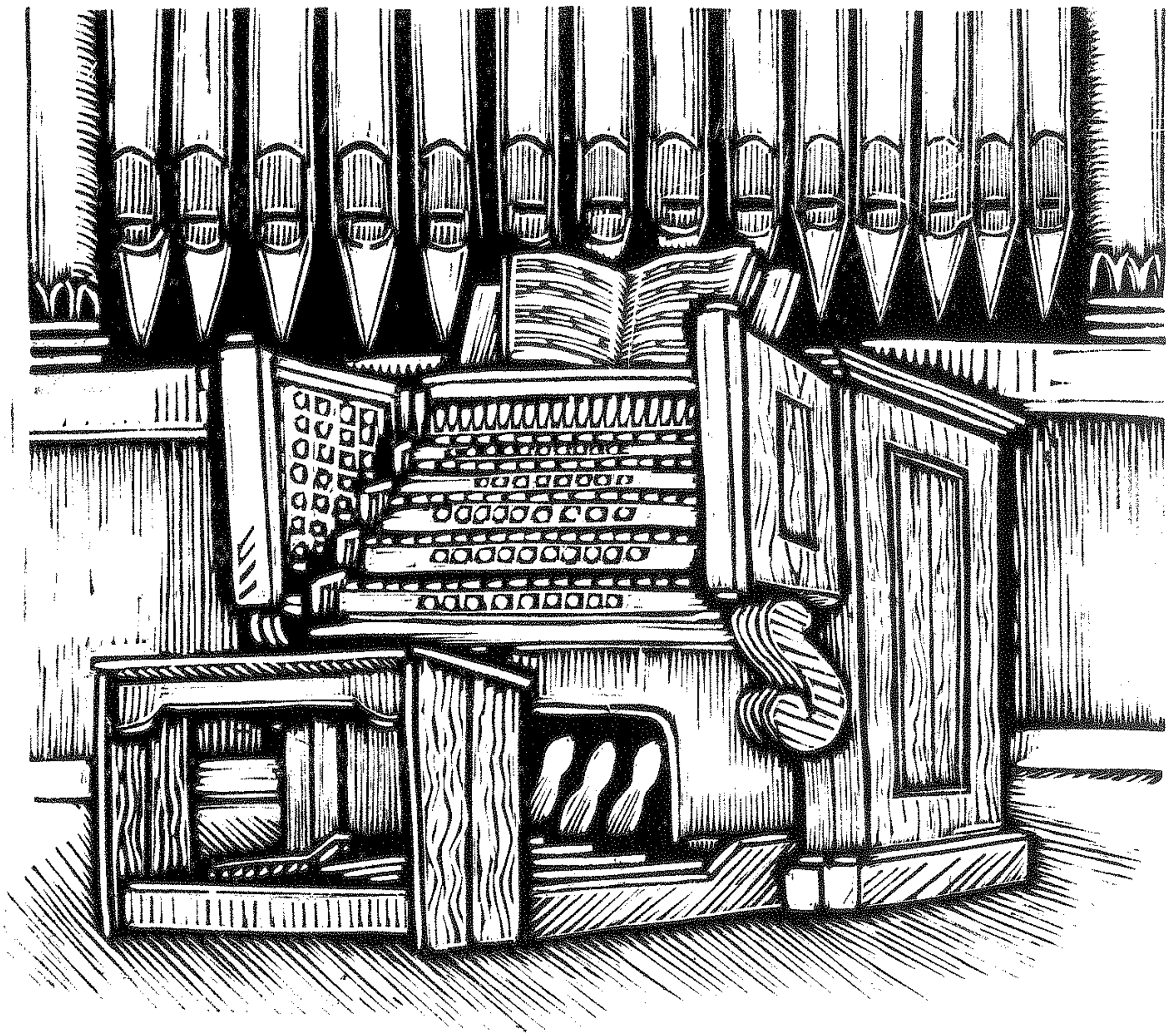


Figure 7. Beating patterns observed for the "football" mode $_0S_2$ after the 1977 Indonesian earthquake at Rarotonga. Because of the Earth's rotation, this mode is split into five singlets with slightly different amplitudes. Interference among the singlets produces this pattern.



have shown that this parameter can be recovered from an inversion of normal mode data acquired from IDA stations.

With the development of instant satellite telecommunications, it should be possible to obtain seismic data from many stations around the world in real time and compute ultralow-frequency source parameters, including seismic moment and rupture orientation, within two or three hours after a major earthquake. Although warnings provided by such computations would not spare epicentral areas, they should permit people living along distant coasts to prepare for tsunamis without having to depend on epicentral reports, which may be unavailable because of local destruction.[19,22]

The parts of this puzzle have yet to be assembled. When that is done, mankind's ancestral interest in music will have yielded an unexpected benefit for society.

References

1. H. Lamb, "The vibration of an elastic sphere," *Proceedings, London Mathematical Society* **13**, pages 189-212 (1882).
2. A. E. H. Love, *Some Problems in Geodynamics* (an essay to which the Adams Prize in the University of Cambridge was adjudged in 1911), Cambridge University Press (1911), 180 pages.
3. H. Benioff, "Long waves observed in the Kamchatka earthquake of November 4, 1952," *Journal of Geophysical Research* **63**, pages 589-593 (1958).
4. C. L. Pekeris and H. Jarosch, "Free oscillations of the Earth," *International Series of Monographs on Earth Science* (E. Ingerson, editor) **1**, pages 171-192 (1958).
5. R. A. Wiggins, "A fast, new computational algorithm for free oscillations and surface waves," *Geophysical Journal of the Royal Astronomical Society* **47**, pages 135-150 (1976).
6. N. Jobert, "Excitation des oscillations propres de torsion de la Terre," *Annales de Geophysique* **19**, pages 372-383 (1962).
7. M. Saito, "Excitation of free oscillations and surface waves by a point source in a vertically heterogeneous Earth," *Journal of Geophysical Research* **72**, pages 3689-3699 (1967).
8. J. F. Gilbert, "Excitation of the normal mode of the Earth by earthquake sources," *Geophysical Journal of the*

Royal Astronomical Society **22,** pages 223-226 (1970).
9. H. Kanamori, "Synthesis of long-period surface waves and its application to earthquake source studies—Kurile Islands Earthquake of October 13, 1963," *Journal of Geophysical Research* **75,** pages 5011-5027 (1970).
10. H. Kanamori and D. L. Anderson, "Amplitude of the Earth's free oscillations and long-period characteristics of the earthquake source," *Journal of Geophysical Research* **80,** pages 1075-1078 (1975).
11. F. Gilbert and A. M. Dziewonski, "An application of normal mode theory to the retrieval of structural parameters and source mechanisms from seismic spectra," *Philosophical Transactions of the Royal Society of London* **278A,** pages 187-269 (1975).
12. A. M. Dziewonski and F. Gilbert, "Observations of normal modes from 84 recordings of the Alaskan earthquake of March 28, 1964, Part II: Further remarks based on new spheroidal overtone data," *Geophysical Journal of the Royal Astronomical Society* **35,** pages 401-437 (1973).
13. E. A. Okal, "The effect of intrinsic oceanic upper-mantle heterogeneity on regionalization of long-period Rayleigh wave phase velocities," *Geophysical Journal of the Royal Astronomical Society* **49,** pages 357-370 (1977).
14. M. Cara, "Regional variations of higher Rayleigh-mode phase velocities: a spatial-filtering method," *Geophysical Journal of the Royal Astronomical Society* **54,** pages 439-460 (1978).
15. B.-G. Jo and E. A. Okal, "Regional dispersion of first-order Rayleigh overtones and its implication for lateral heterogeneity in the mantle," *Transactions of the American Geophysical Union* **62,** page 948 (1981).
16. E. A. Okal and S. Stein, "Measurement and inversion of overtone spheroidal *Q*'s from IDA records," *Transactions of the American Geophysical Union* **62,** page 947 (1981).
17. D. C. Agnew *et al.*, "International deployment of accelerometers: a network for very long period seismology," *Transactions of the American Geophysical Union* **57,** pages 180-188 (1976).
18. S. Stein and R. J. Geller, "Attenuation measurements of split normal modes for the 1960 Chilean and 1964 Alaskan earthquakes," *Bulletin of the Seismological Society of America* **68,** pages 1595-1611 (1978).
19. H. Kanamori and J. W. Given, "Use of long-period surface waves for rapid determination of earthquake-source parameters," *Physics of the Earth and Planetary Interiors* **27,** pages 8-31 (1981).
20. K. Abe, "Size of great earthquakes of 1837-1974 inferred from tsunami data," *Journal of Geophysical Research* **84,** pages 1561-1568 (1979).
21. A. Ben-Menahem and M. Rosenman, "Amplitude patterns of tsunami waves from submarine earthquakes," *Journal of Geophysical Research* **77,** pages 3097-3128 (1972).
22. S. N. Ward, "Relationship of tsunami generation and an earthquake source," *Journal of Physics of the Earth* **28,** pages 441-474 (1980). ■