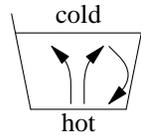


8.5 CONVECTION

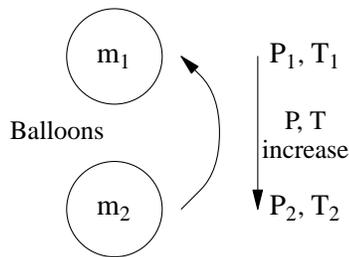
By convection we mean a motion of material due to buoyancy forces resulting from temperature differences.



hot material is less dense than cold material. It rises

The theory of convection is too complicated to get into here, but we may still understand the basic ideas. The first one we need is that of an *adiabatic temperature gradient* or *adiabat*. Despite the scary name its pretty simple.

Suppose in some material, not necessarily a fluid, temperature and pressure increase with depth, and consider two equal masses m_1 and m_2 , each at a different temperature and pressure. Now, magically bring m_2 up to the site m_1 . What happens - m_2 is now a *lower pressure* and so *expands* and *cools* off.



Three possibilities for this scenario:

- 1) If m_2 cools to the same temperature as m_1 the temperature gradient is *adiabatic*.
- 2) If m_2 cools to a temperature above that of m_1 the gradient is *superadiabatic*.
- 3) If m_2 cools to a temperature below that of m_1 , the gradient is *subadiabatic*

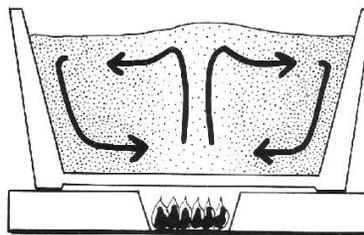
What difference does this make?

For the superadiabatic case m_2 is hotter than the surrounding material, even after the cooling resulting from the pressure change: *so it will continue to expand*. As it expands its density decreases, so it will be less dense than the surrounding material and, thus, tend to rise.

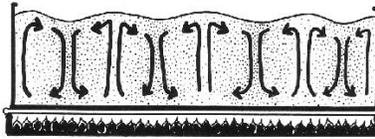
In contrast, for an adiabatic gradient m_2 will come to the same density as the surrounding material and not have any tendency to rise. For a subadiabatic gradient m_2 will be denser than the surrounding material and tend to sink.

Now we see when a material that has a temperature gradient in it is unstable. For example, it is *not* true that "hot air rises" - hot air rises only if the temperature gradient is superadiabatic! An adiabatic temperature gradient is stable - a superadiabatic one isn't and will start to convect if the conditions are right.

In a simple case, the convection pattern is not complicated (the definition of simple) - consider fluid heated from below.



A simple example



A more complex (and more realistic) example

The material on the bottom in the center rises, and material along the bottom is sucked in by the lower pressure. At the top the material cools and it sinks at the sides. This is the most basic *convection cell*

How can we tell if convection will occur? One important factor is how "gooey" the fluid is - easier to get water to boil than maple syrup. Somehow, maple syrup is harder - called Viscosity η . *Viscosity* is how hard it is to shear a liquid

The units are Poise = $\frac{\text{g}}{\text{cm s}}$ or Pascal - s or (Pa-s) = kg/(m s)

For some common materials the values are:

air	10^{-4} poise
water	10^{-2} poise
olive oil	10^{-1} poise
glycerine	20 poise
the mantle	10^{22} poise

Another thing that matters is how much a material expands in response to temperature changes: given by the *coefficient of thermal expansion*, α

$$\frac{dV}{dT} = V\alpha$$

where V is volume and T is the temperature.

So for a large α the material expands a lot for a given temperature change.

Using the previous parameters, the crucial factor for the onset of convection is the Rayleigh number

$$Ra = \frac{\alpha \Delta T g \rho d^3}{\eta \kappa}$$

where:

g is gravity,

ρ is density,

κ is the thermal diffusivity ($= \frac{k}{\rho C_p}$),

d is the depth of the material,

and ΔT is the difference across d in

excess of the adiabatic difference

(since adiabatic difference doesn't cause instability).

At this point we may rewrite the equation as

$$Ra = \frac{\alpha \Delta T g \rho d^3 \rho C_p}{\eta k}$$

using the thermal diffusivity $\kappa = \frac{k}{\rho C_p}$

If the Rayleigh number is greater than about 1500 (depends on the geometry of the experiment) convection will occur. The Rayleigh number is the ratio of factors favoring convection to those opposing it. For example - high temperatures and coefficients of thermal expansion favor convection while high values of diffusivity or viscosity retard

it.

For the solid mantle (gross approximations)

$$g = 10 \text{ m/s}^2$$

$$\alpha = 2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

$$\rho = 3.5 \times 10^3 \text{ kg/m}^3$$

$$d = 3 \times 10^6 \text{ m}$$

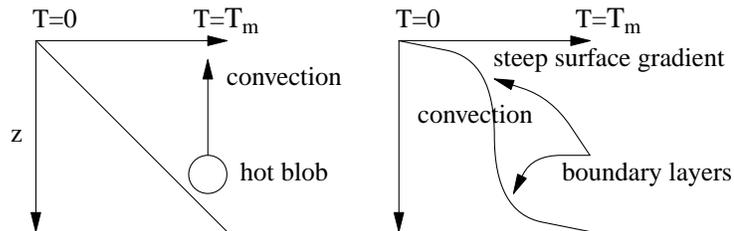
$$k = 10^{-6} \text{ m}^2/\text{s}$$

$$\nu = 10^{21} \text{ Pa}\cdot\text{s}$$

$$\text{Ra} \sim 2 \times 10^4 \Delta T$$

so if the mantle has a superadiabatic temperature gradient we would expect convection.

What does convection do: it transfers heat more efficiently than conduction

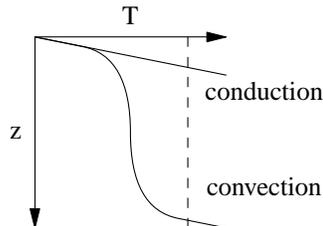


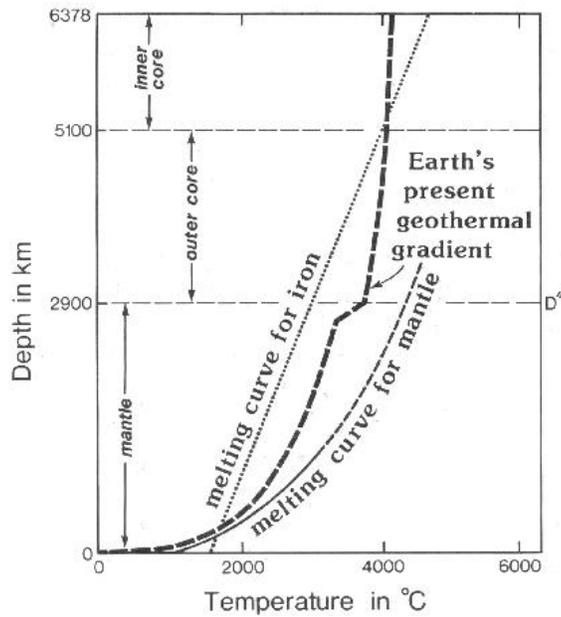
recall: steady state for conduction (if no heat sources) get linear gradient as function of depth.

Convection - brings hot stuff up to the surface - changes geotherm

To be precise - this convection geotherm is an average over the convection cell at each depth. *Note:* This is why Kelvin's calculation went wrong! At the surface a convection geotherm looks like a young conduction geotherm - the convection is constantly resetting the geotherm by bringing up hot material.

This observation also explains how we may avoid having the whole mantle melting, as the conduction geotherm implies.





On earth (as we'll discuss in detail later), plate tectonic process is a surface manifestation of convection. Ridges (spreading centers) mark upwelling, hot limbs. Trenches (subduction zones) mark cold downwelling limbs. This process is more complicated than a simple fluid because the plates form a cold, rigid, lithosphere - the outer boundary layer of convection.

Why? The strength of a rock (how much stress it can support) depends exponentially on temperature.

$$\sigma \sim e^{(-aT)}$$

so the strength drops off fast with temperature and depth.

