HyMaTZ: A Python Program for Modeling Seismic Velocities in Hydrous Regions of the Mantle Transition Zone

Fei Wang¹, Mitchell Barklage¹, Xiaoting Lou¹, Suzan van der Lee¹, Craig R. Bina¹, and Steven D. Jacobsen¹

¹Department of Earth and Planetary Sciences, Northwestern University, Evanston, IL, USA

Abstract

Mapping the spatial distribution of water in the mantle transition zone (MTZ, 410- to 660-km depth) may be approached by combining thermodynamic and experimental mineral physics data with regional studies of seismic velocity and seismic discontinuity structure. HyMaTZ (Hydrous Mantle Transition Zone) is a Python program with graphical user interface, which calculates and displays seismic velocities for different scenarios of hydration in the MTZ for comparison to global or regional seismic-velocity models. The influence of water is applied through a regression to experimental data on how H₂O influences the thermoelastic properties of (Mg,Fe)₂SiO₄ polymorphs: olivine, wadsleyite, and ringwoodite. Adiabatic temperature profiles are internally consistent with dry phase proportion models; however, modeling hydration in HyMaTZ affects only velocities and not phase proportions or discontinuity structure. For wadsleyite, adding 1.65 wt% H₂O or increasing the iron content by 7 mol% leads to roughly equivalent reductions in Vₛ as raising the temperature by 160 K with a pyrolite model in the upper part of the MTZ. The eastern U.S. low-velocity anomaly, which has been interpreted as the result of dehydration of the Farallon slab in the top of the lower mantle, is consistent with hydration of wadsleyite to about 20% of its water storage capacity in the upper MTZ. Velocity gradients with depth in absolute shear velocity models are steeper in all seismic models than all mineralogical models, suggesting that the seismic velocity gradients should be lowered or varied with depth and/or an alternative compositional model is required.

Plain Language Summary

Olivine and its high-pressure polymorphs, wadsleyite and ringwoodite, constitute at least half of the material in the Earth’s upper mantle. In addition to magnesium, iron, silicon, and oxygen, these phases are known for their ability to incorporate H₂O. Water is incorporated into their crystal structures as OH (hydroxyl) defects, charge-balanced primarily by cation vacancies. Hydration of olivine, wadsleyite, and ringwoodite through such defects modifies their density and elastic properties such as the bulk (K) and shear (G) moduli, which can lead to changes in seismic velocities as determined from analyses of seismograms using methods such as seismic tomography. This paper presents a tool for researchers to calculate and graph the influence of hydration in olivine, wadsleyite, and ringwoodite for comparison to any seismic-velocity model. While Hydrous Mantle Transition Zone accounts for elastic property changes due to hydration, the effect of water on thermodynamic phase proportions is not yet included. The goal of such studies is to assess the amount and regional distribution of water in the mantle transition zone.

1. Introduction

Seismic tomography is an imaging technique used to map three-dimensional variations in the seismic-velocity structure of the Earth’s interior at global (e.g., French & Romanowicz, 2014; Grand, 2002; Ritsema et al., 2004; Van der Hilst et al., 1997) or regional scales (e.g., Barklage et al., 2015; Bedle & Van der Lee, 2009; Burdick et al., 2017; Schmandt & Lin, 2014; Van der Lee & Frederiksen, 2005). Regional variations in velocity structure are always relative to one-dimensional seismic velocity profiles that change only with depth. Interpretations of observed seismic velocity variations require mineralogical models of the Earth’s mantle obtained by phase equilibria and thermoelastic properties of major minerals as a function of pressure, temperature, and composition (Goes & Van der Lee, 2002; Li & Liebermann, 2007; Stixrude & Lithgow-Bertelloni, 2012; Wang et al., 2015; Wood, 1995; Xu et al., 2008).

The pyrolite model of the upper mantle and transition zone is composed of ~60% by volume of (Mg,Fe)₂SiO₄ polymorphs, olivine, wadsleyite, and ringwoodite (Irifune & Ringwood, 1987; Ringwood & Major, 1967). These phases are nominally anhydrous minerals (NAMs) but may contain up to several weight percent of H₂O as
hydroxyl defects in their structure (Bolfan-Casanova, 2005; Kohlstedt et al., 1996; Smyth, 1987, 1994). Incorporation of water into NAMs, accompanied by Mg$^{2+}$ point defect vacancies, strongly influences their elastic properties (e.g., Jacobsen, 2006; Mao & Li, 2016) suggesting that the velocity structure of the mantle transition zone (MTZ) may be used to model mantle water content (Thio et al., 2016; Van der Lee et al., 2008).

Whereas some studies suggest efficient dehydration of downgoing slabs in the mantle (e.g., Green et al., 2010), the finding of 1.5 wt% H$_2$O in a natural ringwoodite inclusion in diamond (Pearson et al., 2014) provides evidence that regions of the MTZ where some superdeep diamonds may derive are very hydrous. Regional-scale seismic studies suggest variable water content of the MTZ (Courtier & Revenaugh, 2006; Emry et al., 2015; Liu et al., 2016; Schmandt et al., 2014; Van der Lee et al., 2008; Van der Meijde et al., 2003); however, recent viscosity measurements of hydrated ringwoodite suggest a globally saturated transition zone (Fei et al., 2017). Evaluating the regional-scale water content of the MTZ is important to understanding Earth’s deep water cycle (Hirschmann, 2006; Jacobsen & Van der Lee, 2006; Karato, 2011; Ohtani et al., 2004).

Although the thermodynamic properties of hydrous phases in the deep mantle, especially dense hydrous magnesium silicates, have been thoroughly investigated (e.g., Frost, 2006, Komabayashi & Omori, 2006, Ohtani et al., 2001), the influence of water on phase equilibria in NAMs is largely experimental (e.g., Chen et al., 2002, Frost & Dolejsi, 2007, Smyth & Frost, 2002). Thermodynamic databases for calculating phase equilibria treat H$_2$O as a chemical component in hydrous phases but do not account for water in NAMs (Connolly, 2005; Cottaar et al., 2014; Holland et al., 2013). On the other hand, numerous laboratory studies have been conducted on the influence of water on elastic properties of olivine (Jacobsen et al., 2008; Mao et al., 2010), wadsleyite (Chang et al., 2015; Mao et al., 2008, 2011), and ringwoodite (Jacobsen & Smyth, 2006; Mao et al., 2012; Wang et al., 2006).

In this work, we develop a graphical user interface for a computational tool called Hydrous Mantle Transition Zone (HyMaTZ), written in Python, to evaluate the influence of water on predicted seismic velocities in the MTZ. Phase proportions for various compositional models are calculated using Perple_X (Connolly, 2005) along adiabatic temperature gradients specified by the user. If a dry compositional model is chosen, the thermodynamic data from Perple_X are applied. However, if one provides a water content depth profile, experimental constraints on the influence of H$_2$O on elastic properties are applied through a regression to the provided experimental laboratory database. One can also input a custom variation of elastic properties and density with water content. At this stage, because there is no thermodynamic database for H$_2$O in NAMs, the phase proportions used by HyMaTZ are not affected by water. In this way, HyMaTZ allows one to explore the influence of water on bulk compressional ($V_P$) and shear wave velocity ($V_S$) in the depth range of ~300–700 km for comparison to regional or global seismic velocity models. In this paper, some example uses of HyMaTZ are shown, and it is demonstrated that increasing the water content of the upper MTZ by 1.65 wt% H$_2$O in wadsleyite has about the same influence on $V_S$ as increasing the iron content by ~7 mol% or raising the temperature by 160 K.

2. Methods

2.1. Thermoelastic Properties

Three independent physical parameters, namely, density ($\rho$), adiabatic bulk modulus ($K_S$), and shear modulus ($G$), are required to calculate the seismic velocities

$$V_P = \sqrt{\frac{K_S + \frac{4}{3}G}{\rho}}$$

$$V_S = \sqrt{\frac{G}{\rho}}$$

where $V_P$ and $V_S$ are the P wave and S wave velocities, respectively. In order to calculate the density, bulk modulus, and shear modulus of a polimineratic rock, one requires elastic properties of all the individual components, either experimentally or computationally derived, an equation of state (EOS) that describes the variation of elastic properties with pressure and temperature, and a polycrystalline averaging scheme that
distributes stress or strain uniformly across all grains. In addition, the compositional sensitivity of thermoelastic properties requires thermodynamic or experimental input. In this work, the thermoelastic properties of all the anhydrous minerals are derived from the output of Perple_X from a variety of referenced compositional models.

2.1.1. Internally Consistent Thermodynamic Data Set

In order to calculate the thermoelastic properties of minerals at mantle conditions, a thermodynamic database and equations of state appropriate for the desired range of pressures and temperatures are essential. An assessment of phase equilibria calculations for mantle rock compositions as a function of pressure and temperature using Perple_X gives consistent results with observations (Connolly, 2009). The work of establishing a self-consistent database fitted to many different types of thermodynamic experiments has been an iterative process. The database published by Stixrude and Lithgow-Bertelloni (2011) was developed by requiring that all data satisfy self-consistency of fundamental thermodynamics relationships, and it is appropriate for use in the current work because the data set provides a rigorous baseline for dry compositions.

2.1.2. Equation of State

The high P-T conditions of the transition zone require the use of a thermodynamic EOS to interpolate and extrapolate the available experimental data to those conditions. For HyMaTZ, we have chosen to use the third-order Birch-Murnaghan formalism with the Mie-Grüneisen-Debye temperature correction developed by Stixrude and Lithgow-Bertelloni (2005). This has also been shown to give good results in calculating phase equilibria in the Earth’s mantle (Stixrude & Lithgow-Bertelloni, 2011).

To calculate the thermoelastic properties of each mineral across P-T conditions of the MTZ, a total of 11 parameters are used in the EOS model (Stixrude & Lithgow-Bertelloni, 2005, 2011). Referring to equation (1), the adiabatic modulus \( K_S \) is used, whereas the internally consistent thermodynamic data set uses the isothermal bulk modulus, \( K_T \). We obtain \( K_S \) from \( K_T \) using the following relation:

\[
K_S = K_T \left(1 + \gamma \alpha T \right)
\]

where \( \gamma \) is the Grüneisen parameter and \( \alpha \) is the coefficient of thermal expansion, related by

\[
\alpha = \gamma \frac{C_v}{K_T} \rho
\]

where \( \rho \) is density and \( C_v \) is the isochoric heat capacity. Detailed explanation of the EOS and its application are given in Stixrude and Lithgow-Bertelloni (2005) and by Cottaar et al. (2014).

2.1.3. Mineral Phase Proportion Models

HyMaTZ uses the output of Perple_X (Connolly, 2005) for obtaining thermodynamic quantities and phase proportions of the minerals at mantle conditions. Perple_X implements the EOS described above and Gibbs-free energy minimization to calculate the equilibrium mineral phases as a function of chemical composition, pressure, and temperature. Six oxides are used to define the bulk chemical composition: CaO, FeO, MgO, Al₂O₃, SiO₂, and Na₂O, which can comprise 47 phases (Stixrude & Lithgow-Bertelloni, 2011). Once the user specifies the foot temperature at the Earth’s surface for an adiabatic temperature profile, HyMaTZ accesses an included data file containing the phase equilibria (mineral proportions and compositions) as well as the thermodynamics variables (e.g., entropy and density) at each P-T point on a 2-D grid with the pressure range of 0–30 GPa and from 773–2773 K. The model is composed of 600 nodes on the pressure axis and 400 nodes on the temperature axis. For the purpose of plotting a more precise phase proportion model, parameters at P-T points in between nodes are obtained by linear interpolation. Because mantle temperatures above ~300-km depth are not adiabatic and are highly region dependent, the results of HyMaTZ are most valid for the 300- to 700-km range.

HyMaTZ includes data files from Perple_X for four compositional models: pyrolite, basalt, and harzburgite from Xu et al. (2008) and piclogite from Weidner (1986). Phase proportions and thermodynamic data are extracted from these data files along the user-specified adiabat (section 2.1.4). A list of the compositional models included in HyMaTZ is shown in Table 1. In addition to these models, HyMaTZ includes fixed percentage mixtures of harzburgite and basalt at 90/10, 80/20, 70/30, 60/40, 50/50, 40/60, 30/70, 20/80, and 10/90 of harzburgite/basalt, where the ratios give their molar compositional proportions input to Perple_X. In addition to these models composed of equilibrated compositional mixtures of harzburgite and basalt, the user can
work with a nonequilibrated mechanical mixture of any two end-member models in any percentage. By mechanical mixing we mean that the output phase proportions and thermodynamic properties are averaged according to the volume proportions of each, specified by the user.

2.1.4. Pressure and Temperature

In preparation for running a velocity model calculation with HyMaTZ, we include the capability to calculate self-consistent pressures and isentropic temperatures in a 1-D column in the Earth, which is divided into 300 nodes. The calculation begins at the first node by calculating the density at 1 bar and at a foot temperature provided in step 2 by the user specifying the isentropic potential temperature at the Earth’s surface. The pressure-depth profile along the user-specified adiabat is obtained iteratively with

$$dP = \rho(r) g(r) dr$$

where \( P \) is pressure, \( \rho \) is the average density between two nodes, \( g \) is the acceleration due to gravity, and \( r \) is radius from the Earth’s center. The next pressure is calculated by initially taking the density at the next node to be the same as at the previous node, calculating the overburden pressure (equation (5)), and then updating the pressure to calculate the next value of density. From the pressure, temperature is determined from the output of Perple_X (included in the Models folder) using the requirement that the two nodes have the same entropy. Finally, the process is iterated until the density difference between two consecutive iterations is less than 0.01 kg/m³, which usually converges in two or three steps, as previously reported by Afonso et al. (2008).

2.1.5. Averaging Schemes for Polycrystalline Rocks

For a polyminerlic rock, there is no exact formulation to calculate its bulk elastic properties (e.g., Bina & Helffrich, 1992). Averaging schemes, which distribute stress, strain, or strain energy uniformly across all grains, are provided as options in HyMaTZ. The density of the assemblage is calculated using

$$\rho = \sum V_i \rho_i$$

where \( \rho \) is the whole rock density and \( V_i \) and \( \rho_i \) are the volume fraction and density of each mineral constituent in the rock. For the bulk and shear moduli, Voigt’s assumption (Watt et al., 1976) is that the total strain is uniform throughout the rock

$$M_v = \sum V_i M_i$$

where \( M_v \) is the whole-rock Voigt bulk or shear modulus and \( V_i \) and \( M_i \) are the volume fraction and bulk or shear modulus of each constituent mineral. The Reuss assumption (Watt et al., 1976) is that the stress field remains constant throughout the rock

$$\frac{1}{M_R} = \sum V_i \frac{1}{M_i}$$

where \( M_R \) is the whole-rock Reuss bulk or shear modulus and \( V_i \) and \( M_i \) are the volume fraction and bulk or shear modulus of each constituent mineral. A commonly used approach is to average the Voigt and Reuss bounds, which gives the Voigt-Reuss-Hill average (Watt et al., 1976)

$$M = \frac{M_R + M_v}{2}$$

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**Table 1**

*End-Member Compositional Models Included With HyMaTZ*

<table>
<thead>
<tr>
<th>Model</th>
<th>Reference</th>
<th>SiO₂</th>
<th>MgO</th>
<th>FeO</th>
<th>Na₂O</th>
<th>Al₂O₃</th>
<th>CaO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyrolite</td>
<td>Xu et al. (2008)</td>
<td>44.93</td>
<td>38.83</td>
<td>8.57</td>
<td>0.13</td>
<td>4.36</td>
<td>3.18</td>
</tr>
<tr>
<td>Harzburgite</td>
<td>Xu et al. (2008)</td>
<td>43.52</td>
<td>45.73</td>
<td>8.76</td>
<td>0</td>
<td>1.09</td>
<td>0.91</td>
</tr>
<tr>
<td>Basalt</td>
<td>Xu et al. (2008)</td>
<td>50.39</td>
<td>9.76</td>
<td>8.22</td>
<td>2.19</td>
<td>16.84</td>
<td>12.61</td>
</tr>
<tr>
<td>Piclogite</td>
<td>Weidner (1986)</td>
<td>48.91</td>
<td>21.01</td>
<td>5.69</td>
<td>3.00</td>
<td>14.39</td>
<td>7.00</td>
</tr>
</tbody>
</table>
In another averaging scheme, Hashin and Shtrikman (1963) assume that the strain energy remains constant throughout the rock, with upper bound

\[ K_{HS} = K_{MAX} + \frac{V_i}{(K_i - K_{MAX})^{-1} + V_{MAX}(K_{MAX} - \frac{4}{3}G_{MAX})^{-1}} \tag{10} \]

\[ G_{HS} = G_{MAX} + \frac{V_i}{(G_i - G_{MAX})^{-1} + \frac{2V_iK_{MAX}G_{MAX}}{5G_{MAX}(K_{MAX}^2 + G_{MAX})}} \tag{11} \]

where \( K_{MAX} \) and \( G_{MAX} \) are the largest bulk and shear moduli among minerals in the rock and \( V_i, K_i \) and \( G_i \) are the bulk volume fraction, bulk modulus, and shear modulus of each constituent mineral. When instead of
they are modified by H2O, which are also summarized in Table 2. The equations displayed on the graphs are as follows for each phase:

**Olivine:**

\[
\begin{align*}
\rho_0 &= 3227.3(4.1) - 53.3(11.8) C_{H2O} + 1178.9(40.6) X_{Fe} \\
K_{SO} &= 129.8(0.6) - 8.4(2.0) C_{H2O} + 5.9(1.8) X_{Fe} \\
G_0 &= 81.9(0.4) - 41.8(1.2) C_{H2O} - 31.8(1.8) X_{Fe}
\end{align*}
\]

**Wadsleyite:**

\[
\begin{align*}
\rho_0 &= 3475.3(3.3) - 48.8(3.3) C_{H2O} + 1179.6(48.7) X_{Fe} \\
K_{SO} &= 170.4(0.7) + 8.7(0.7) C_{H2O} + 24.1(8.2) X_{Fe} \\
G_0 &= 112.7(0.7) - 4.8(0.5) C_{H2O} - 44.0(8.7) X_{Fe}
\end{align*}
\]
In equations (13)–(21), the numbers in parentheses are 1σ uncertainties in the corresponding coefficients. Some of the uncertainties in the fitted coefficients shown in equations (13)–(21) are large. This is due to the fact that there are few data points along with scatter in the data. We conducted error analysis with the Monte Carlo (statistical) method (Connolly & Khan, 2016), showing that the propagated errors in seismic velocities are less than 1% (Figure 2). The reason for this is because the physical properties were mainly determined by the a coefficient (constant term) in the correlation, and the uncertainty for that is small. Conversely, the large uncertainties in the b coefficients, which is the slope along the water concentration axis, indicate that there would be a large uncertainty when variations in the seismic velocity are used to estimate the water concentration by inversion. The error analysis results shown in Figure 2 give the distributions in the seismic velocities and density when the coefficients in equations (13)–(21) are treated as random variables with a normal distribution. The calculation used the 1600 K foot temperature and 50% WSC pyrolite model.

2.3. Water Content Depth Profiles

Olivine, wadsleyite, and ringwoodite have different WSC, which is the maximum water content in the solid phase without stabilizing a coexisting hydrous melt or fluid (Hirschmann et al., 2005). The water storage capacity of (Mg,Fe)2SiO4 polymorphs varies with pressure, temperature, oxygen fugacity, and bulk (rock) mineral assemblage (e.g., Bolfan-Casanova et al.,...
2012; Demouchy et al., 2005; Kohlstedt et al., 1996; Tenner et al., 2012). For simplicity, HyMaTZ works with a user-defined water content profile, which refers to variation in the amount of water in the (Mg,Fe)2SiO4 phases as a function of depth. To create the default water content depth profile, we take storage capacity for wadsleyite to be 3.3 wt% (Smyth, 1987) and then apply partition coefficients of 6:30:15 for the ratios of water in olivine: wadsleyite: ringwoodite from Inoue et al. (2010). The water content profile of olivine as a function of depth was taken from the experimental data of Hauri et al. (2006). To produce different water content profiles through the upper mantle and transition zone, the user specifies the percentage of storage capacity for each phase. For example, if the user specifies 50% WSC for their profile, wadsleyite will have 1.65 wt% H2O (constant) and ringwoodite will have half that or 0.825 wt% H2O (constant). The partition coefficient between olivine: wadsleyite from Inoue et al., 2010 is 6:30, so in the above example (50% WSC), the olivine at 410-km depth would have 5 times less water than the wadsleyite or 0.33 wt% H2O. The water content profile of olivine as a function of depth still follows the depth profile given in Hauri et al. (2006), but it is anchored to 0.33 wt% H2O at 410-km depth. We note that the partitioning of water between wadsleyite and olivine here is a simplistic approach and does not account for partitioning of water between olivine and the full pyrolitic assemblage (Tenner et al., 2012) nor can it account for the production of partial melting when the storage capacity of one phase is exceeded across the phase boundaries. HyMaTZ also allows the user to provide a custom water content profile by specifying in equation form how the water content varies with depth for each phase of (Mg,Fe)2SiO4. If the user does not select to modify elastic properties with hydration from the regression fittings, rather than using the dry density and elastic moduli obtained from the regression, the thermoelastic properties from the Perple_X output will be applied.

2.4. Anelasticity

The velocities calculated by equations (1) and (2) neglect the effect of internal friction and hence are independent of frequency. However, it is known that some velocity dispersion and attenuation of seismic waves occur, and theoretical calculations and experimental work (Karato, 1993; Karato & Spetzler, 1990) have shown that the underlying anelasticity can also decrease seismic velocities. Karato and Jung (1998) showed that water can significantly reduce seismic wave velocities through anelastic effects. In HyMaTZ, one has the option to correct the calculated mineral acoustic velocities with anelasticity models derived from long-period seismic data analysis by the following equations (Goes et al., 2000; Minster & Anderson, 1981):

\[ Q_s = Q_p = A \omega \exp\left(\frac{\rho H}{RT}\right) \]  

(22)

\[ H = E + PV \]  

(23)

\[ Q_p^{-1} = (1 - L)Q_s^{-1} + LQ_\mu^{-1} \]  

(24)

\[ L = \left(\frac{4}{3}\right) + \left(\frac{V_s}{V_p}\right) \]  

(25)

\[ V_{(P,T,X,\omega)} = V_{(P,T,X)} \left(1 - \frac{Q_\mu^{-1}(\omega, T)}{2 \tan\left(\frac{\pi a}{2}\right)}\right) \]  

(26)

where \( A \) is a constant, \( R \) is the gas constant, \( H \) is activation enthalpy, and \( E \) is activation energy, and equation (22) is based on a thermodynamic (transition state theory) correlation of relaxation times versus temperature and pressure. \( V \) is activation volume, \( \omega \) is the seismic frequency, \( a \) is the frequency exponent in the power law model of \( Q \) and \( 0 \leq a \leq 1 \), and \( P \) and \( T \) are pressure and temperature. \( Q_s \) and \( Q_p \) are the \( Q \) values for \( P \) and \( S \) waves, and \( Q_s \) and \( Q_\mu \) are the \( Q \) values for the bulk and shear moduli. In the mantle, \( Q_\mu > Q_s \), and is usually taken to be infinity. HyMaTZ also contains attenuation models by Cammarano et al. (2003) and Goes et al. (2000). The user can also input a custom attenuation model.

3. Results

3.1. Benchmarking

Running HyMaTZ consists of several steps: (1) selecting a compositional model, (2) calculating phase proportions along a user-selected adiabatic temperature gradient, (3) applying a water content profile,
(4) choosing how water influences thermoelastic properties of olivine polymorphs, and finally (5) selecting a polycrystalline averaging scheme. To test the validity of our approach against previous work, we provide comparisons to some existing programs for the case of a dry MTZ. Figure 3 shows a comparison of HyMaTZ output with the output from the BurnMan code (Cottaar et al., 2014) for pure forsterite, Figure 3. Comparison of output from HyMaTZ (green curves) and BurnMan (blue open circles; Cottaar et al., 2014) for $V_p$, $V_s$, and density of pure forsterite along a 1600-K adiabatic temperature gradient. HyMaTZ = Hydrous Mantle Transition Zone.

Figure 4. Comparison of velocities obtained for the 1600-K isentropic pyrolite model from HyMaTZ with those computed by Stixrude and Lithgow-Bertelloni (2011). HyMaTZ = Hydrous Mantle Transition Zone.
Mg$_2$SiO$_4$. At all conditions between 200- and 800-km depth, the difference between HyMaTZ and Burnman velocities for Mg$_2$SiO$_4$ is less than $5 \times 10^{-6}$%.

Next we compare the output of HyMaTZ to predicted seismic velocities from Perple_X using the database of Stixrude and Lithgow-Bertelloni (2011). Because HyMaTZ uses the same EOS in Perple_X, this benchmark tests our usage of the implementation of the EOS. Figure 4 shows a typical comparison with data provided by Stixrude and Lithgow-Bertelloni (2011) for a 1600-K isentropic pyrolite model. We observe a difference of up to $\pm 0.5\%$ in $V_p$ and $V_s$, mainly at the major phase transition depths (410, 520, and 660 km), which may be due to two reasons. First, although Perple_X produced phase proportions very close to those of Stixrude and Lithgow-Bertelloni (2011), there were still some small differences in phase proportions that can lead to $0.5\%$ difference in the seismic velocities and densities. This difference is possibly because we calculate phase proportions on a 2-D grid and then use interpolation to obtain parameters in between grid points; thus, the phase proportions are not exactly the same as the directly calculated values when the pressure and temperature are not on a grid point. Second, we calculated the pressure and temperature versus depth profiles using the interpolated phase proportions; thus, the differences in the phase proportions will propagate to the calculated pressures and temperatures. This was most obvious in the region of the seismic discontinuities, that is, the largest differences of up to 1% between HyMaTZ velocities and those of Stixrude and Lithgow-Bertelloni (2011) were around the 410-, 660-, and 730-km phase boundaries. A slight difference in the $P$-$T$ node conditions can cause a small shift in the depth of the phase transition. Overall, HyMaTZ produces velocity profiles for dry pyrolyte that are broadly consistent with Burnman and Perple_X using the Stixrude and Lithgow-Bertelloni (2011) database.

### 3.2. A Sample Calculation

As an example of how the program is run, we go through a HyMaTZ calculation using the pyrolite model (Xu et al., 2008) for a 1600-K adiabatic temperature profile. Detailed instructions are provided in the downloadable software manual. Figure 5 shows the HyMaTZ dashboard. Along the left-hand side of the dashboard, HyMaTZ displays a numbered list of instructions that are followed stepwise: (1) select a compositional model, (2) set the adiabatic foot temperature at the surface, (3) display the phase proportion model, (4) indicate whether a dry or hydrous model will be used for (Mg,Fe)$_2$SiO$_4$ phases, and (5) define the water content profile. In step 6, the user can alter the forsterite number (Fo#) of (Mg,Fe)$_2$SiO$_4$ polymorphs, in which case the values

![Figure 5. Screenshot of the HyMaTZ dashboard. A set of instructions appear along the left-hand side of the dashboard. At right, the phase proportions and velocities are plotted, along with a comparison to the PEMC model (Dziewonski et al., 1975). HyMaTZ = Hydrous Mantle Transition Zone.](image-url)
of density, \( K_{\text{SO}} \), and \( G \), are derived from the fitted regressions in step 4. However, if the user customizes the Fo\# in the olivine polymorphs, it should be clarified that there will be no effect on the phase proportions, which were fixed in step 1. Instead, this variable is only meant to allow one to test the comparative effects of changing Fe and H\(_2\)O on velocities. In step 7, a polycrystalline averaging scheme is selected, and in step 8 the user can select an attenuation model, if desired. In step 9, the velocities are calculated, and in step 10 the user can select what to plot, such as \( V_p \), \( V_s \), \( V_p/V_s \), temperature, bulk modulus, shear modulus, or density. In step 11, one can add a reference model for comparison, including PREM (Dziewonski et al., 1975), ak135 (Kennett et al., 1995), IASP91 (Kennett & Engdahl, 1991), or PEM-C (Dziewonski et al., 1975). In addition, the user can input a custom velocity profile here, which, for example, could be a user-generated regional velocity model.

Figure 6 further illustrates how the results of a velocity calculation are displayed. In this example we calculate velocities through a pyrolite model (Xu et al., 2008) with a 1600-K adiabatic temperature profile and a mantle of pyrolite composition (Xu et al., 2008).

3.3. Trade-Offs in Parameters Influencing Seismic Velocity

Temperature is the most sensitive parameter in calculating seismic velocities because it strongly influences both phase proportions and thermoelastic variables. Several studies have focused on how the temperature affects seismic velocities (Goes et al., 2000; Goes & Van der Lee, 2002), suggesting that just 100-K difference could influence \( V_s \) by \( \approx 1\% \) in the shallow mantle and by 0.5% in the lower transition zone (Cammarano et al., 2003). Composition is another sensitive factor in the calculation of seismic velocities. Cammarano et al. (2003) performed calculations with varied Fe, Al, and Ca contents and found that an increase in the Fe content from 7% to 11% gave decreased \( V_s \) by 1% and \( V_p \) by 0.7%. Xu et al. (2008) suggested that the mantle may be heterogeneous, being better represented by a mechanical mixture of basalt and harzburgite, and they showed that the \( V_s \) in a mechanical-mixture transition zone is faster and increases more rapidly with depth. Accordingly, in HyMaTZ, we provide several options that let the user examine how mechanical mixtures of compositional models change the calculated seismic velocity profiles. Figure 7 shows an example of how temperature, water, and Fe content trade off in the calculated velocity profile of pyrolite in the transition zone. Compared with a dry MTZ, increasing the water content in wadsleyite to 1.65 wt% (50% WSC), increasing the temperature by 160 K, or increasing the Fe content from Fo 90 to Fo83 (increase of \( X_{\text{Fe}} = 0.07 \)) produce comparable reductions in seismic velocity for the pyrolite model.

Whereas HyMaTZ modifies \( \rho \), \( K_{\text{SO}} \), and \( G \) with hydration through a regression to experimental data, the pressure derivatives of these parameters are taken to be unchanged from those in the thermodynamic database, parameterized according to the Birch-Murnaghan EOS. However, the user can also input customized pressure derivatives to test the influence of.
K_0 = (dK/dP)_{P=0} \text{ and } G_0' = (dG/dP)_{P=0}. \text{ In Figure 8, the outcome for several combinations of } K_0' \text{ and } G_0' \text{ is plotted, which may be influenced by water content. For example, whereas Chang et al. (2015) show that water does not strongly influence } K_0' \text{ of wadsleyite, elasticity data for hydrous ringwoodite suggest elevated } K_0' \text{ and } G_0' \text{ (Jacobsen & Smyth, 2006; Smyth et al., 2004; Thio et al., 2016; Wang et al., 2006; Ye et al., 2012). In Figure 8, velocities of dry pyrolite are shown when the } K_0' \text{ and } G_0' \text{ of ringwoodite are varied to illustrate the influence of the pressure derivatives.}

### 3.4. Influence of Water on the Seismic Velocity in the Transition Zone: Comparison With a Regional Model

Van der Lee and Nolet (1997) used regional waveform tomography to image mantle structure beneath the North American continent. Their results showed that in the MTZ beneath the eastern U.S. continental margin shear wave velocities reach to over 1% lower than the continental average, confirming earlier hints for such a deep low-velocity region (Grand, 1994; Romanowicz, 1979). Van der Lee et al. (2008) showed that in 3-D tomographic model NA04 of Van der Lee and Frederiksen (2005), the East Coast anomaly extends from transition zone depths up to depths around 100 km beneath the U.S. East Coast. The anomaly is also present in more recent tomographic models such as NA07 (Bedle & Van der Lee, 2009) and SL2013NA (Schaeffer & Lebedev, 2014) which includes USArray data. Van der Lee et al. (2008) argued that a changed temperature profile is not an obvious explanation for the anomaly because there is neither an obvious heat source (such as a mantle plume) in tomographic models nor do independent observations of discontinuity characteristics or surface heat flow suggest elevated heat (Goes & Van der Lee, 2002; Pollack et al., 1993). At least one study shows evidence for an uplifted 410-km discontinuity and a strengthened 520-km discontinuity beneath the eastern United States (Courtier & Revenaugh, 2006), suggesting that hydration of the upper mantle provides an alternative explanation.

We used three different water content depth profiles that simulate a dry mantle, a hypothesized intermediate mantle condition, and a maximally wet mantle, and we synthesized their velocity profiles, illustrated in Figure 9. Profile 1 (red line) is for the case of a dry mantle. Profile 2 (green line) is constructed for 20% WSC (i.e., 0.13 wt% H_2O in olivine at 410 km, 0.65 wt% H_2O in wadsleyite, and 0.33 wt% H_2O in ringwoodite). Profile 3 (blue line) is for the case where the WSC is maximum (i.e., 0.66 wt% H_2O in olivine at 410 km, 3.3 wt% H_2O in wadsleyite, and 1.7 wt% H_2O in ringwoodite). Two regional S wave velocity profiles were constructed as the 1-D average through a portion of 3-D model NA04 (Van der Lee & Frederiksen, 2005), of which one averages the eastern U.S. low-velocity anomaly region and one represents an average from outside the anomaly region. For all three water content profiles, calculated velocities in the uppermost mantle are similar to or lower than the reference seismic profile from outside the anomaly. The three profiles are similar to the seismic profile from within the low-velocity anomaly between 280 and 340 km, but their gradients are less steep than the seismic profile’s gradient.

While hydration could affect the velocities in NA04 at depths shallower than 400 km, uncertainties in the velocity gradient of the seismic reference model preclude an unambiguous interpretation of the absolute velocities. The transition zone exhibits more variations in the HyMaTz profiles as a result of hydration. Figure 9 suggests that the uppermost part of the transition zone beneath eastern North America may be moderately hydrous.
since the dry model (red curve) matches velocities outside the eastern U.S. low-velocity region and the model constructed with 20% water storage capacity (green curve) compares well to the 1-D average profile within the eastern U.S. low-velocity anomaly, as previously suggested by Van der Lee et al. (2008). However, comparison of absolute velocities across the entire depth region of the MTZ shows significant differences between the HyMaTZ calculations and the velocities from seismic models, mostly, and as above, as a result of the HyMaTZ reference velocity model having a less steep gradient than the reference seismic profile’s gradient. Removing the seismic reference velocities from the anomalous velocities and removing the dry HyMaTZ velocities from the other HyMaTZ velocity profiles allow us to compare velocity anomalies (Figure 10b). Figure 10a shows the differences of the profiles in Figure 9b with respect to the reference seismic model (black dotted line), which allows us to visualize error bars reflecting uncertainties in the HyMaTZ profiles estimated above and uncertainties in the seismic profiles estimated by Bedle and Van der Lee (2009) from an ensemble of tomographic models with good data fits. These tomographic models are guided to depart minimally from a reference model and also have a limited range of smoothness constraints, likely leaving the uncertainties underestimated. Figure 10b shows that the green water content profile fits the seismic profile for the anomaly well, within uncertainties, over the entire depth range of the transition zone, suggesting that a mantle with olivine polymorph water content of 20% of the storage capacity is a reasonable explanation for the anomaly.

4. Conclusions

1. HyMaTZ is a freely available Python code with graphical user interface used to calculate seismic velocity profiles for various mineralogical models where the elastic properties of (Mg,Fe)2SiO4 polymorphs may be varied by effects due to hydration, calculated from a regression of experimental data.

2. At conditions of the uppermost transition zone, increasing the water content in wadsleyite to 1.65 wt% H2O (50% WSC) causes an equivalent reduction in S wave velocity in pyrolite as increasing the iron content by 7 mol% or raising the temperature of a 1600-K pyrolite model by 160 K.

3. The eastern U.S. low-velocity anomaly is consistent with wadsleyite and ringwoodite in pyrolite containing 0.65 and 0.33 wt% H2O (20% WSC), respectively, but the absolute velocity of the lower MTZ is faster than all HyMaTZ models, suggesting that the pressure derivatives of ringwoodite should be elevated (consistent with hydration), the seismic-velocity gradient should be reduced and/or vary with depth in the seismic model, or an alternative compositional model is required to better fit the deeper MTZ.

References


